Optical manipulation of atomic motion for a compact gravitational sensor with a Bose-Einstein condensate interferometer

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Abstract

Atom interferometers are among the best available devices for gravitational sensing. Standard devices, although unrivaled in sensitivity, cannot be made compact because they require the atom packets to be in free-fall over large distances on the order of one meter. In our experiments we create a novel type of interferometer as a proof-of-concept for a compact gravitometer. The limitation of a large drop distance is overcome by repeatedly applying a pulsed optical lattice to suspend two vertically separated packets of ultra-cold $^{87}$Rb atoms while keeping them in a state of virtual free fall.

To be competitive with the sensitivity of previous devices, many optical pulses will be required. An in-depth experimental and theoretical study of our pulses was performed. Previous methods could reflect the atomic motion with a theoretical maximum fidelity of 0.94, which would unacceptably limit the number of pulses that could be applied. This motivated the development of new high-fidelity manipulation pulses based on the idea of intensity pulse shaping. Several new pulse sequences of various orders were created and tested. Theoretical simulations predict fidelities that differ from unity by less than 1 part in $10^4$ and experiment verified that fidelities are in fact greater than 0.99. With these pulses a single cloud of atoms was suspended against gravity for more than 150 ms by bouncing the atoms with 130 consecutive pulses. Previous bouncing experiments using other methods have only demonstrated a maximum of 3 bounces. Furthermore, this represents an unprecedented transfer of 260 single-photon momenta.

A vertically oriented atom interferometer using ultra-cold atoms was implemented, requiring only 10 $\mu$m of vertical drop distance. After multiple pulses, the packets are recombined and an interference signal is observed. 81 successive operations were
applied for a total interferometer time of nearly 50 ms. This work marks the first
time so many individual pulses have been used in an atom interferometer of any kind.
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In recent years research involving Bose-Einstein condensation (BEC) has matured past the point of merely studying the properties of this new state of matter to the stage where it can be utilized in practical devices that could have a strong impact on everyday life. Among the possible uses are measurement devices based on the principles of interferometry. In general an interferometer is a device that measures a phase induced between two traveling waves. A common example of an interferometer can be found on most commercial airliners and uses interfering light waves from...
a coherent laser source as the reference for an inertial guidance system. These are known as optical Sagnac interferometers and are used as rotation sensors for navigation. But interferometers are not restricted to use interfering electromagnetic waves; interference can be observed from any type of coherent waves.

In 1924 Louis de Broglie generalized the wave-particle duality, introduced by Einstein for the photon, to massive particles and gave the formula

$$\lambda = \frac{h}{p}$$

relating the wavelength $\lambda$ to $p$ the momentum of the particle, where $h = 2\pi\hbar$ is Plank’s constant. This concept led people to consider matter-wave interference. Shortly after, in 1927, Davisson and Germer observed the interference of electrons through diffraction from a the surface of a metallic crystal. Diffraction of neutron beams followed. In 1975, Colella et al. [1] demonstrated the operation of a neutron interferometer.

### 1.1 Motivation

Over the past decade techniques have been developed to manipulate the motion and quantum state of atoms with exquisite control. One significant achievement of this emergent field, called atom optics, has been the development of interferometers [2, 3, 4, 5] based on the wave properties of atoms described by quantum mechanics. These interferometers differ from the traditional optical interferometers in that they use cold atoms instead of light. For atoms the phase of the interferometer is given by the Bohr phase $Et/h$ where $E$ is the energy of the atom and $t$ is the measurement time. Because it is very easy to change the energy of an atom, these interferometers offer the promise of being much more sensitive than their optical counterparts to certain effects. Among these are inertial effects such as rotation and acceleration, making
atom interferometers very useful in sensors for navigation and other such devices.

A related line of interest is the development of atom interferometry for measuring $g$, the acceleration due to gravity, and already atom interferometers are among the best available gravitometers [6]. The acceleration of gravity, denoted as $g$, is a location-dependent quantity whose measurement reveals information about nearby mass. Since the pull of gravity diminishes as one moves farther from the center of the earth, a highly sensitive gravitometer could be used to detect small changes in elevation. In fact, an accuracy of one part in 10 billion could measure a change in altitude to the nearest half of a millimeter. This would be useful for seismology to monitor Earth movements. Additionally, a portable device with this level of sensitivity would enable geologists to detect changes in the composition of buried rock formations by measuring gravitational differences, since gravity would be a bit stronger over higher-density formations. This technique is used to search for underground oil and mineral deposits or monitor changes in the water table.

1.1.1 Atom verses photon

To illustrate the increased sensitivity of atom interferometers over optical interferometers, let us consider both types in a hypothetical situation to measure the pull of gravity at the surface of the Earth. The sensitivity of each device is proportional to the phase that the atom or photon gains over some vertical displacement which in turn, is proportional to the change in energy of the atom or photon. The wavelength of a photon near the surface of the Earth traveling vertically away is changed slightly due to the gravitational redshift. The fractional change in the photon’s wavelength is given by [7]

$$\Delta z = -\frac{GM}{c^2 r^2} \Delta r,$$  \hspace{1cm} (1.2)
where $G$ is the gravitational constant and $c$ is the speed of light, while $M$ and $r$ are the Earth’s mass and radius respectively. This shift in wavelength is equivalent to a loss of the photon’s energy,

$$\Delta E_{\text{photon}} = \hbar \Delta \omega = -\frac{2\pi \hbar c}{\lambda} \Delta z.$$  \hspace{1cm} \text{(1.3)}

On the other hand, for a $^{87}\text{Rb}$ atom, the change in energy at the surface of the Earth due to a vertical displacement would be

$$\Delta E_{\text{atom}} = mg\Delta r,$$  \hspace{1cm} \text{(1.4)}

where $m$ is the mass of $^{87}\text{Rb}$, $g$ is the acceleration of an object due to Earth’s gravity at the surface, and $\Delta r$ is the vertical displacement. The ratio of change in energy

$$\frac{\Delta E_{\text{atom}}}{\Delta E_{\text{photon}}} = \frac{\lambda cr^2 mg}{2\pi \hbar GM} \approx 10^{11}. \hspace{1cm} \text{(1.5)}$$

gives the ratio of phase accumulated and represents an incredible advantage in sensitivity for atoms over photons. Unfortunately it is generally much harder to work with atoms. Furthermore, photons possess the advantage of strength in numbers. The overall phase sensitivity of the interferometer is limited by the number of particles because of shot-noise. The noise in the phase scales like $\frac{1}{\sqrt{N}}$ where $N$ is the number of particles. For a lower noise floor, it is advantageous to use a large number of particles and typical photon production rates exceed that of even the best thermal atom beam interferometer experiments by a factor of $10^{10}$. Because the phase is proportional to the measurement time, long times can help us overcome this shortcoming. For devices of equal path length, the measurement time for atoms exceeds that of photons by a factor of $3 \times 10^{11}$. Measurement times of about one second [6, 8] for atom interferometers can be expected. This makes up some ground that was lost due to production
rates and still leaves a possibility for the sensitivity of an atom gravitometer to exceed that of a purely optical interferometer. This example illustrates the theoretical advantage in sensitivity of atoms over photons and provides the motivation for pursuing atom interferometry, for gravimetry.

1.1.2 Compact atom gravimetry

In general, atom interferometry for measuring gravity requires that the atoms be in free fall and as such, standard devices are arranged in a fountain geometry where the atoms are tossed up and allowed to follow a ballistic trajectory. Because long measurement times are needed for high sensitivity, the free fall requirement dictates that large drop distances are also required. This seems to contradict the notion of having a small portable device with enough sensitivity to do precision gravimetry. An example of the large size constraint is illustrated in figure 1.1.2 by an actual atom gravitometer. Over the last few years engineering has progressed allowing for smaller packages for cold and ultra-cold atom sources. Among these advances are chip-based BEC production techniques where the atom trap uses micro-fabricated wires for magnetic confinement [9]. One of the main advantages to the chip traps are their small size. Unfortunately, these advances do not alleviate the large drop distance required for the fountain geometry and does little to shrink the overall size of the device. As chip-based BEC production methods continue to advance, it becomes especially desirable to find a way around this limitation.

The technique implemented in this thesis overcomes the seemingly necessary large size requirement. Through repeated application of a laser field designed to reflect the motion of a condensate, we find it is possible to keep two packets of ultra-cold atoms suspended, while still allowing them to be in effective free-fall. After the desired “hang-time” the phase between the packets can be detected and a value of $g$ calcu-
lated. Additionally, low atomic velocity enables precise control of the motion of the condensate with a pulsed optical lattice and gives a higher contrast output signal than can be expected from thermal or cold atom interferometers. To be competitive with the fountain devices, hundreds or thousands of manipulation pulses will be required depending on the interferometer geometry. In this work, we demonstrate that many (>100) pulses can be used together to control the kinetics of ultra-cold atoms, providing packet suspension for times greater than 150ms.

1.2 Format of this dissertation

The remaining chapters detail the implementation of atom interferometry with ultra-cold atoms for gravitational sensing. Some of the work presented here builds upon
that presented in the theses of Reeves [10], Garcia [11], and Deissler [12].

The format of this dissertation is as follows. Chapter 2 gives a brief overview of the procedure and portion of our experimental apparatus devoted to BEC production. The main emphasis is on creating the BEC as a source of ultra-cold atoms for interferometry. Although the capability of producing BEC is still far from being common-place, BEC production with this apparatus has been thoroughly covered by others and will be addressed in this thesis by a short review. Additional details may be found in other theses [10, 11, 12]. Chapter 3 describes the method of using momentum transfer from a pulsed standing wave to manipulate the motion of atoms. Included is a detailed study of the pulses that we have previously used for BEC interferometry, high-lighting the development of new high-fidelity pulses based on intensity pulse shaping. Chapter 4 describes how we applied these new pulses to bounce a free-falling condensate, dynamically suspending it against gravity for 150 ms. By optimizing the timing between pulses, we are able to extract a value for $g$. Additionally, using the bouncing technique, we were able to demonstrate a Stern-Gerlach experiment that was sensitive to $\mu$T/m field gradients. In chapter 5, the significance of all these previous developments culminate in the achievement of two different configurations of vertical interferometers where the atoms only drop about 10 $\mu$m.
Rubidium Snow Cones:  
A source of ultra-cold atoms for matter-wave interferometry

The overall goal of this work is to exploit the wave properties of massive particles to perform vertical interferometry for measuring $g$. This has been motivated thus far by pointing out that gravity affects matter to a higher degree than it affects light. Perhaps not adequately motivated is why we wish to use ultra-cold $^{87}$Rb atoms instead of some other massive object. After all isn’t coherence of the wave the main requirement for achieving interference? The answer to this question is a resounding yes, and as explained below, therein lies the justification.
The coherence length $L$ is a quantity used with lasers to quantify the degree in which the beam can interfere with itself sometime $t_c = L/c$ later. The coherence of a laser is related to the spectral width $\Delta f$ or equivalently $\Delta \lambda$ by

$$L = \frac{c}{\Delta f} = \frac{\lambda^2}{\Delta \lambda} \quad (2.1)$$

where $c$ is the speed of light in vacuum. We can relate this to the coherence of a matter wave. As a collection of particles is cooled, the average velocity of each particle will decrease and the de Broglie wavelength

$$\lambda_{db} = \frac{h}{p} = \frac{h}{mv} \quad (2.2)$$

where $m$ is the mass, will increase. Replacing $\lambda$ with $\lambda_{db}$ from (2.1), we can say that the coherence of a collection of particles goes like

$$\sim \frac{\lambda_{db}^2}{\Delta \lambda_{db}} \quad (2.3)$$

An increase in $\lambda_{db}$ however is not sufficient for an increase in coherence. But cooling the collection not only decreased the average velocity $v$, it also decreases the spread in velocities $\Delta v$. Differentiating (2.2) we find the spread in wavelength

$$\Delta \lambda_{db} = \frac{h}{mv^2} \Delta v. \quad (2.4)$$

Substituting (2.2) and (2.4) into (2.3) shows that the coherence of the matter wave goes like

$$\sim \frac{h}{m \Delta v}. \quad (2.5)$$

Ultra-cold atoms, have a very narrow velocity spread and give us the level of coherence
that we seek. Techniques for cooling to very low temperatures have been developed for creating Bose-Einstein condensates [13] with $^{87}\text{Rb}$. This elemental choice was largely due to the convenient laser transitions of $^{87}\text{Rb}$, the ability to confine it in a magnetic trap, and good collisional properties.

2.1 Ultra-cold atom source: BEC

Bose-Einstein condensation (BEC) is a quantum phase transition that occurs in bosons at very low temperatures and high densities. The theory behind BEC started in 1920 with the work done by a physicist from India, Satyendra Nath Bose, who devised a statistical method for counting states of indistinguishable photons. Five years later, Albert Einstein applied Bose’s method to massive particles with integer spin (bosons) and found that at a critical value in temperature, a macroscopic fraction of the particles will occupy the same quantum state. This phenomenon was first experimentally demonstrated [13] nearly 70 years later by University of Colorado physicists, Eric Cornell and Carl Wieman, with the production of the first Bose-Einstein condensates Fig. 2.2. The critical temperature for a 3D gas containing non-interacting bosons is given by,

$$T_c = \left( \frac{n}{\zeta(3/2)} \right)^{2/3} \frac{h^2}{2\pi mk_B},$$  \hspace{1cm} (2.6)

were $n$ is the density, $\zeta$ is the Riemann zeta function, $h$ is plank’s constant, $m$ is the mass, and $k_B$ is Boltzmann’s constant. Equivalently, the condition for condensation can be expressed as a critical phase space density $n\lambda^3$ where $n\lambda^3 \geq 2.61$.

We will create a BEC as our source of ultra-cold atoms in our interferometry experiments. The experiments are performed in a closed chamber under ultra-high vacuum conditions. The vacuum system consists of two chambers linked by a thin tube and is differentially pumped. A schematic of the experimental setup can be seen
Figure 2.1: Schematic of the experimental setup. Two glass vacuum chambers are linked by a thin tube. In the MOT chamber, atoms are loaded into a MOT before being loaded into a quadrupole magnetic trap. The coils generating the quadrupole field are moved using a translation stage towards the science chamber, which has a better vacuum. The atoms are then transferred into a TOP trap, with a rotating bias field generated by the waveguide structure.

Figure 2.2: Absorption images of atomic clouds during stages of evaporation; thermal cloud, condensate surrounded by thermal cloud, pure condensate.

in Fig. 2.1. We start by making a magneto-optical trap (MOT) in the MOT chamber, where the vacuum pressure is around $2 \times 10^{-9}$ Torr. The atoms are then transferred to a magnetic trap and moved to the science chamber, where the pressure is around $3 \times 10^{-11}$ Torr. There, the atoms are evaporatively cooled down to the condensation temperature.
2.1.1 Magneto-optical trap

The process of creating a BEC begins by collecting room temperature $^{87}\text{Rb}$ atoms in a magneto-optical trap (MOT). The MOT incorporates laser cooling and a magnetic field gradient to capture about $4 \times 10^9$ atoms and cool them to $250 \mu\text{K}$. The $^{87}\text{Rb}$ atoms are released into the MOT chamber by running $2 - 3$ A of current through a pair of commercially available getters. The atoms caught in a magneto-optical trap [14], consisting of three pairs of perpendicular laser beams tuned slightly red of the $5^2S_{1/2}, F = 2 \rightarrow 5^2P_{3/2}, F = 3$ transition of $^{87}\text{Rb}$ as well as a smaller amount of light tuned to the $5^2S_{1/2}, F = 1 \rightarrow 5^2P_{3/2}, F = 2$ transition as “repump” light. The magnetic field is a quadrupole field generated by two coils in an anti-Helmholtz configuration with a field gradient of $10 \text{G/cm}$. With a MOT, we get a phase space density $\rho = n\lambda^3_{\text{DB}}$ of $n\lambda^3 \approx 5 \times 10^{-7}$. Due to the increased opacity as the MOT is loaded, cooling light becomes unable to penetrate the core of the cloud and further cooling is hindered. Although this is already very cold and quite dense, the phase space density is still several orders of magnitude away from the required $n\lambda^3 \geq 2.61$. To go any further, we are required to extinguish the cooling lasers and use another method to support and cool the atoms. Evaporative cooling to BEC has been demonstrated in both magnetic traps and off-resonant optical dipole traps [15]. In our experiment, we use a magnetic trap.

2.1.2 Magnetic trap

Magnetic traps take advantage of the magnetic moments of neutral atoms and the forces exerted on these by inhomogeneous magnetic fields. Rubidium has one unpaired electron that can give rise to an overall magnetic dipole moment. The energy of an atomic level with angular momentum $\vec{F}$ and magnetic quantum number $m_F$ in a
magnetic field of strength $B$ is [16]

$$E(m_F) = g\mu_B m_F B \equiv \mu B,$$  \hspace{1cm} (2.7)

where $g$ is the gyromagnetic moment and $\mu_B$ the Bohr magneton. The potential that the atom sees will be proportional to the strength of the magnetic field. If we use this to trap and cool atoms, the depth of the trap will be on the order of $T = \mu (B_{\text{max}} - B_{\text{min}})/k_B$, where $T \ll 1 \, \text{K}$ for magnetic fields that are conveniently generated. The atoms must therefore be pre-cooled to $\mu \text{K}$ temperatures by the MOT in order to be loaded into a magnetic trap for further cooling. From Eq. (2.7), we see that if $g m_F > 0$, we have a weak-field seeking state that needs a minimum in $B$ in order to be trapped, while for $g m_F < 0$, we have a strong-field seeking state, being trapped at a maximum in $B$. It can be shown that no local maxima of $B$ are possible, so only weak-field seeking states can be trapped [17]. In our case, we trap the $|F = 2, m_F = 2\rangle$ atoms in the $5S_{1/2}$ ground state. For our magnetic trap, we create the quadrupole field with two current coils arranged around the vacuum chamber in an anti-Helmholtz configuration. Supplying a maximum of 750 Amps to these coils gives a gradient of about 450 G/cm and provides purely magnetic confinement. We initially load atoms into the quadrupole trap, then after an initial step of evaporative cooling, the atoms are transferred to a time-orbiting potential (TOP) trap.

**Quadrupole trap**

The quadrupole trap consists of two identical coaxial coils carrying currents in opposite directions (anti-Helmholtz coils) as in the case of the MOT coils. A simple configuration is an axially symmetric trap, which gives a linear potential with

$$\hat{x} \cdot \frac{d\vec{B}}{dx} = \hat{y} \cdot \frac{d\vec{B}}{dy} = -2\hat{z} \cdot \frac{d\vec{B}}{dz},$$  \hspace{1cm} (2.8)
where the condition $\nabla \cdot B = 0$ from Maxwell’s equations requires the factor of 2 in the $\hat{z}$ term. The coordinates are defined in Fig. 2.1. The quadrupole trap is generated by a magnetic field

$$\vec{B}(\vec{r}) = B'_x (2z\hat{z} - x\hat{x} - y\hat{y}),$$

(2.9)

which gives a potential

$$U = \mu |\vec{B}| = \mu B'_x \sqrt{4z^2 + x^2 + y^2}.$$ 

(2.10)

The quadrupole trap has one major problem. Atoms can undergo a spin flip at a zero point of the magnetic field, a so-called Majorana spin flip, and thus fall out of the trap [18]. In a quadrupole trap, these losses are especially important because the zero of the magnetic field is in the center of the trap, where the density of atoms is the highest, and becomes all the more important at lower temperatures, when the density in the center of the trap increases.

**TOP trap**

Though the implementation of a quadrupole trap is extremely simple, losses from Majorana spin flips will limit its usefulness at lower temperatures. Adding a spatially uniform time-independent bias field to this arrangement does not eliminate the zero point, but simply shifts it, so a more elaborate setup is needed. What is desired is, in effect, a potential with a minimum that is non-zero. This can be achieved by adding a time-dependent bias field

$$\vec{B}_0 = B_0 (\hat{x} \cos \Omega t + \hat{z} \sin \Omega t),$$

(2.11)
which is added to the quadrupole field

\[ \vec{B}_q = B'_x (2z\hat{z} - x\hat{x} - y\hat{y}) \]  

(2.12)

to form a time-orbiting potential (TOP) trap [19].

The magnitude of the combined field is then

\[ |\vec{B}| = \left[ B^2_0 + B'^2_x (x^2 + y^2 + 4z^2) - 2B_0B'_x x \cos \Omega t + 4B_0B'_x z \sin \Omega t \right]^{1/2}. \]  

(2.13)

From this equation, we can see that the zero of the magnetic field moves in an ellipse with minor radius \( r_D = B_0/2B'_x \), the radius of the so-called circle of death.

If the oscillation frequency \( \Omega \) of the bias field is chosen such that it is smaller than the Larmor frequency \( \omega_L = \mu B/\hbar \approx 10 \) MHz, the atomic spins can adiabatically follow the field. Choosing \( \Omega \) to be larger than the frequency of atomic motion (about 10 Hz) means that the atoms will not be able to physically follow the zero of the field. They therefore experience a time-averaged potential. Expanding the square root to second order and taking the time average then gives

\[ \langle |\vec{B}| \rangle = B_0 + \frac{B'^2_x}{B_0} \left( \frac{1}{4}x^2 + \frac{1}{2}y^2 + z^2 \right), \]  

(2.14)

which generates a harmonic potential with a minimum of \( \mu B_0 \). This also gives us the trap depth of the TOP trap. Inserting the radius of the circle of death into Eq. (2.14) we get \( U = \mu \langle |\vec{B}(r_D)| \rangle = \frac{1}{2} \mu |B_0| \), since atoms which reach the circle of death are removed from the trap. In our trap, the maximum trap depth is only about 350 \( \mu \)K at a bias field of 20.5 G. We can find the oscillation frequencies in the trap \( \omega_i \) using \( U = \mu \langle |\vec{B}| \rangle \) and Eq. (2.14) to get

\[ \omega^2_x = \frac{1}{2} \frac{\mu B'^2_x}{mB_0}, \quad \omega^2_y = \frac{\mu B'^2_x}{mB_0}, \quad \omega^2_z = 2 \frac{\mu B'^2_x}{mB_0}. \]  

(2.15)
The TOP trap is the trap in which we perform the final stages of evaporative cooling and will make the BEC.

### 2.1.3 Evaporation

After loading the atoms into the magnetic trap, we implement a technique called forced RF evaporative cooling [20] to selectively eject high-energy atoms from the trap. This works by using a small antenna that emits RF. The frequency of the rf is chosen to be resonant with atoms near the edge of the trap and drives spin transitions of $\Delta m = 1$ with $\hbar \omega_{RF} = \frac{1}{2} \mu B_{edge}$. Hot atoms become anti-trapped and are ejected from the edge of the trap while the remaining atoms rethermalize to a lower temperature, just as a hot beverage cools as steam rises from its surface. Rethermalization occurs via interatomic scattering among the remaining atoms in the trap. In the atomic clouds used in our experiment, the atomic density is relatively low, leading to low interatomic scattering rates and long evaporation times of around 30 s. In the MOT chamber, the lifetime in the quadrupole trap is limited by collisions with hot background atoms to around 4 s. Since the lifetimes in the trap must be long compared to the evaporation time, we must move the atoms to region with a lower background pressure. With the quadrupole trap loaded, the coils are moved using a translation stage over a distance of about 0.5 m to the science chamber, where a better vacuum gives lifetimes of around 80 s. During the transfer, most atoms remain in the trap and heating is small. We observe $1.5 \times 10^9$ atoms at 1 mK in the magnetic trap on the science chamber side.

### 2.2 Waveguide

The condensate is adiabatically loaded into a specially designed waveguide [10]. The guide is a four-rod structure as seen in Fig. 2.3 and produces a 2D quadrupole field in
the plane perpendicular to the rods. By modulating the current through these rods in an appropriate manner, very weak confinement can be achieved while still providing support against gravity. Magnetic fields from the current leads give minimal confinement along the axis of the guide. Our waveguide has the following trapping frequencies; $\omega_x = 6$ Hz, $\omega_z = 3.3$ Hz, and $\omega_y = 1.2$ Hz. For previous guided-wave condensate interferometry experiments performed in our lab, the waveguide structure has served a critical role of supporting the atoms and defining the “arms” of the interferometer. The gravitometer relies on free-falling atoms and so the guide is not utilized here during the actual interferometer sequence. Although the gravitometer will not be performed in the waveguide, in part of chapter ??, we use the very controlled environment of the waveguide for testing the operations required for manipulating the motion of the atoms. The other thing that the waveguide offers is a very low temperatures. Adiabatic loading of such a weak trap gives condensate temperatures of $\sim 800$ pK. For this reason, even in the later bouncing experiments of chapter ?? and the gravity experiments of chapter ?? we choose to release the atoms from the weak trap defined by the guide.
Shooting Atoms in a Barrel:
Optical control of atomic motion

In all interferometers, it is important to have a means to split a wave into two or more pieces, direct the paths of each piece, and then coherently recombine the pieces back together to measure a phase. In the case of an optical interferometer, the splitting and recombination are often performed using optical beam splitters while mirrors are employed to direct the path of the light. Just as the high-quality beam splitters and mirrors are necessary elements for manipulating the path of a laser beam in an optical interferometer, successful implementation of an atom interferometer relies on being
able to manipulate the motion of atoms with high precision.

The analog of these two optical elements for atom systems, namely splitting and reflection, have been implemented in several different ways with varying levels of success. Some groups doing interferometry with thermal atomic beams have used material surfaces with an etched periodic structure as a grating to diffract the atomic wave function [21]. Others have used a magnetic wedge in micro-fabricated magnetic waveguides to separate an atomic cloud [22], but this method requires the atom cloud to be very close to the surface of a current-carrying wire where patch potentials have proved to be problematic. Recently groups have been able to use the harmonic potential from a magnetic trap to reflect the motion of the atoms [23]. Although this ”reflect” is essentially 100% efficient and does not introduce any decoherence to the atomic wave function, there are many instances where it would be better to perform interferometry in the absence of a confining potential. Still others have tried reflecting atoms by bouncing them off of blue-detuned light sheets [24] or evanescent waves [25]. Unfortunately, these two methods have proven to be ultra-sensitive to any irregularities in light intensity and surface flatness. Another method that was first used in thermal-beam interferometry and has since proved useful in ultra-cold atom interferometry is the off-resonant standing wave laser beam. This method is the one that we use and has the advantage of being able to precisely impart momentum to the atoms. The interaction of the light field with the atoms during the application of the pulse is most easily understood as absorption of a photon from one beam followed by stimulated emission into the counter-propagating beam. Both a beam splitter and mirror can be constructed with this principle.
### 3.1 Manipulating atomic motion with standing wave lasers

Optical control of atomic motion has developed into a fertile field. One important tool is the off-resonant standing wave laser beam [26, 27]. Atoms in such a beam experience a spatially oscillating potential that can act as a diffraction grating for the atomic wave function in the same way that a conventional grating acts for a light wave. This effect was originally demonstrated by deflecting thermal atomic beams [28, 29, 30], and the technique has seen much use in atom-beam interferometers [31, 21]. More recently, similar effects have been achieved using nearly stationary atoms produced using laser cooling or Bose-Einstein condensation [32, 33]. This too has been applied to atom interferometers [34, 5, 35, 4, 36] and other experiments [37, 38, 39] and is our method of choice for performing the beam-splitting and reflection operations in our condensate interferometer.

#### 3.1.1 Theory

We can approximate the condensate as a plane wave because the atoms in a BEC are coherent and have a very narrow velocity spread. This allows us to calculate its time evolution in the presence of a periodic optical potential using the momentum states of the wave packets. The energy levels of an atom are shifted when exposed to light by an amount proportional to the intensity of the light [26]. This effect is called the light shift or ac Stark shift. Passing a laser beam with intensity $I_0$ through the atoms and using a mirror on the other side to retro-reflect the beam, results in the periodic optical intensity

$$I = I_0 \cos^2(ky + \frac{a}{2}) = \text{const} + \frac{I_0}{2} \cos(2ky + \alpha). \quad (3.1)$$
which is proportional to the potential energy experienced by the atoms. Dropping
the constant term and neglecting atomic interactions and any magnetic confinement,
the one-dimensional Schrödinger equation for atoms in a periodic potential such as
this off-resonant standing wave can be written as

\[
\frac{d\psi}{dt} = \left[ -\frac{\hbar}{2M} \frac{\partial^2}{\partial y^2} + \beta \cos(2ky + \alpha) \right] \psi, \tag{3.2}
\]

where \( \beta \) is proportional to the laser intensity and \( \alpha \) is the phase of the standing wave.

**Split**

We consider the beam-splitting operation first implemented in [5] and the reflection
operation demonstrated in [4]. A general understanding of these manipulations can be
obtained by considering the three states \(|0\rangle, |+v_0\rangle\) and \(|-v_0\rangle\), where the state labels
give the atomic velocity and \(v_0 = 2\hbar k/M\) for atomic mass \(M\) and light wavenumber
\(k\). Taking \(\alpha = 0\), the optical standing wave potential can be expressed as \(\hbar \beta \cos 2ky\).
This couples the state \(|0\rangle\) to the states \(|\pm v_0\rangle\) via matrix elements \(\hbar \beta/2\), driving the
beam-splitting transition

\[
|0\rangle \leftrightarrow |+\rangle \equiv \frac{1}{\sqrt{2}} (|v_0\rangle + |-v_0\rangle). \tag{3.3}
\]

Since the \(|0\rangle\) and \(|+\rangle\) states have energies that differ by \(mv_0^2/2\) the transition is
not resonant and cannot be made with perfect efficiency using a single pulse. An
appropriate pulse can, however, create the superposition \((|0\rangle + |+\rangle)/\sqrt{2}\). After the
pulse, the phase of the superposition evolves in time and the state eventually becomes
\((|0\rangle - |+\rangle)/\sqrt{2}\). A second identical laser pulse applied to this state drives it to the
desired \(|+\rangle\). Each pulse is 24 \(\mu s\) with an intensity \(\beta = 2.80\omega_r\) where \(\omega_r\) is the recoil
frequency. The delay between pules is 33 \(\mu s\). This analysis follows that of Wu *et*
Reflect

The reflection operation $|+v_0\rangle \leftrightarrow |-v_0\rangle$ couples two states of equal energy, so it can be achieved with a single weak pulse. However, in this limit the transition is slow, making the operation very sensitive to the initial atomic velocity. Faster operation can be obtained using a stronger pulse, but the $|0\rangle$ state can then be populated and must be accounted for. To do so, write the state $|+v_0\rangle$ as $(|+\rangle + |-\rangle)/\sqrt{2}$, where the antisymmetric state $|-\rangle = (|+v_0\rangle - |-v_0\rangle)/\sqrt{2}$ has no coupling to the $|0\rangle$ state. The $|-\rangle$ state does, however, acquire an energy shift while the standing wave is applied, making its phase evolve in time. For an appropriate pulse intensity and duration, the $|+\rangle$ branch of the wave function makes two full Rabi oscillations between $|0\rangle$ and $|+\rangle$, ending up back in $|+\rangle$ as it started. At the same time, the $|-\rangle$ branch acquires a $\pi$ phase shift, making the total state $(|+\rangle - |-\rangle)/\sqrt{2} = |-v_0\rangle$ and achieving the desired reflection. Our reflect pulse is 76 $\mu$s long with an intensity of $\beta = 4.28\omega_r$.

3.1.2 Fidelity measurements

Several studies of the interaction between atoms and optical standing waves have been performed in other groups using atomic beams [31, 21], but the spread in atomic velocity complicated comparisons with theory and also limited the fidelity with which operations such as beam-splitting and reflection could be applied. Accurate operations are advantageous for most applications. For instance, in most interferometer schemes, imperfect operations reduce signal visibility through the loss of atoms and the presence of non-interfering atoms in the measured output states. More specifically, for interferometers such as in [5, 4], a stationary condensate is split into two packets moving apart. After some time the packets are brought back together and
returned to zero velocity. Any residual atoms left at zero velocity after the splitting operation would interfere with the final recombination and introduce phase errors. The magnitude of the phase error is on the order of the fraction of atoms left behind by the splitting.

Fortunately, it is possible to achieve high-fidelity operations for ultracold atoms. This has been observed in experiments for some time, but a quantitative study had not been reported. In this section, we investigate the dependence of the operations on two key parameters, the laser intensity and residual atomic velocity, and find good agreement with theoretical expectations. We also observe a level of fidelity that has not been previously reported. This demonstrates the possibility to control atomic motion with precision comparable to that previously achievable only with internal state transitions.

We applied these operations to Bose-Einstein condensates consisting of about $10^4$ $^{87}$Rb atoms held in a magnetic guide. One feature of the guide is its relatively weak confinement [40]: for the experiments described here, the harmonic oscillation frequencies were $\omega_x = 2\pi \times 7.4$ Hz, $\omega_y = 2\pi \times 0.8$ Hz, and $\omega_z = 2\pi \times 4.3$ Hz, with the standing-wave laser beam oriented parallel to the $y$-axis. As a result, the atomic densities are relatively low and interaction effects are negligible.

Another relevant aspect of the apparatus is the procedure by which atoms are loaded into the guide. The condensates are produced in a tighter trap suitable for evaporative cooling. The tight trap and the guide are coincident, and the guide is loaded by slowly ramping up the guide field and then ramping down the tight trap field. During the final ramp, the trap frequencies pass through a value of 60 Hz, at which point ambient fields with a few mG amplitude can excite center-of-mass oscillation of atoms. Velocities of up to 0.5 mm/s were observed as a result, which is large enough to affect the splitting and reflection operations. Removing nearby
noise sources reduced the velocity amplitude to about 0.2 mm/s. Further control was obtained by synchronizing the loading process with the power line frequency and starting the experiment at the turning point of the atomic motion where the velocity was near zero.

Alternatively, we could use the loading process to controllably impart a velocity to the atoms. In this case, the centers of the tight trap and the guide were deliberately offset, and the tight trap current was abruptly switched off before the final ramp was completed. This caused the atoms to oscillate with a large amplitude. By starting the experiment at different points in the cycle, various atomic velocities were sampled and the effect of velocity on the standing-wave operations was investigated. Since the operations require less than 1 ms to perform, the atomic velocity was essentially constant during the experiment.

The standing wave was produced by a home-built diode laser. The laser was tuned 12.8 GHz blue of the $5S_{1/2}$ to $5P_{3/2}$ atomic transition, at a wavelength of 780.220 nm. A single acousto-optic modulator (AOM) was used to turn the beam on and off and another one to adjust the intensity of the beam. The output of the AOM was coupled into a single-mode fiber which provided spatial filter and pointing stability. The fiber output had a power of up to 12 mW and an approximately Gaussian profile with beam waist of 0.7 mm. The standing wave was generated by passing the beam through the vacuum cell and retro-reflecting it using an external mirror. The cell was constructed with vacuum windows that were anti-reflection coated on both sides. We found this to be critical: The cell used in Ref. [4] had uncoated windows and the multiple reflections produced a speckle pattern in the reflected beam with intensity variations of over 50%. This variation made it difficult to achieve consistent results.

The experiments were performed by applying one or more pulses of the standing wave and then allowing the atoms to evolve freely for 30 ms so that packets with
different velocities would separate in space. The atoms were observed by passing a resonant probe beam through them into a camera, with the resulting images showing the positions of the packets and the number of atoms in each. The images were analyzed by fitting the packets to Gaussian profiles. For faint packets, the widths of the profiles were fixed to match those observed for packets containing many atoms, typically 10 $\mu$m and 70 $\mu$m in the $x$ and $y$ directions, respectively.

In general, several packets are observed in a given image. The packets can be labeled according to their velocity index $n$, defined by $v = v_i + nv_0$ where $v$ is the observed velocity and $v_i$ the initial velocity. The fraction of atoms in each packet is denoted $N_n$. The accuracy of the operation is quantified using the fidelity, defined as

$$F = |\langle \psi_0 | \psi \rangle|^2$$

for desired state $\psi_0$ and observed state $\psi$. For instance, an ideal splitting operation produces the state $|+\rangle$, so if the actual state is

$$|\psi\rangle = \sum_n c_n |v_i + nv_0\rangle$$

then the fidelity would be

$$F_{\text{split}} = \frac{1}{2} |c_{+1} + c_{-1}|^2 = \frac{1}{2} \left( N_{+1} + N_{-1} + 2\text{Re}c_{+1}^*c_{-1} \right).$$

In general this depends on the phase difference between $c_{+1}$ and $c_{-1}$, which of course cannot be determined by simple imaging. However, interferometer operation does not depend on this phase as long as it is constant, since it can be absorbed into the definitions of the states $|\pm v_0\rangle$ without changing the ultimate results. For this reason,
we ignore phase effects and evaluate the fidelity as

\[ F_{\text{split}} = \frac{1}{2} \left( N_{+1} + N_{-1} + 2\sqrt{N_{+1}N_{-1}} \right). \]  \hfill (3.7)

Split

We observe the most accurate splitting operation for zero-velocity atoms and a standing-wave laser power of 0.34 mW. Fig. 3.1 shows the results obtained as the power and velocity were varied around these values. Each data point represents a single measurement. Repeated measurements under the same conditions showed variations of about 0.01 in \( N_0 \). The optimal parameters yielded fidelities of about 0.995, with repeated measurements consistently above 0.99. It was possible to
maintain split fidelities at this level for several hours before adjustments were needed to compensate for long-term experimental drifts.

The theoretical curves shown in Figs. 3.1 and 3.2 are calculated by solving the Schrödinger equation numerically in the presence of the optical potential.

\[
i \frac{d\psi}{dt} = \left[ -\frac{\hbar}{2M} \frac{\partial^2 \psi}{\partial y^2} + \beta \cos(2ky) \psi \right],
\]

(3.8)

using the Bloch expansion

\[
\psi(y, t) = \sum_n c_n(t) e^{i(2nk+\kappa)y},
\]

(3.9)

where \(\kappa = Mv_i/\hbar\) [35]. The coefficients \(c_n\) satisfy

\[
i \frac{dc_n}{dt} = \frac{\hbar}{2M} (2nk + \kappa)^2 c_n + \frac{\beta}{2} (c_{n-1} + c_{n+1}).
\]

(3.10)

We solved equations (3.17) numerically, including indices \(n\) up to \(\pm 4\). (Negligible population was observed in the \(c_{\pm 4}\) states.) We applied the initial condition \(c_0 = 1\) and calculated the fidelity as described above.

To compare with the experimental results, it was necessary to relate \(\beta\) to the measured beam power. We treat the atoms as two-level systems, in which the \(5S_{1/2}F = 2\) ground hyperfine state is coupled to the \(5P_{3/2}\) excited state. We neglect the excited state hyperfine splitting, since it is small compared to the laser detuning \(\Delta\). The optical potential is then given by [26]

\[
V(y) = \frac{\hbar \Gamma^2 I(y)}{8\Delta I_{\text{sat}}}
\]

(3.11)

where \(\Gamma = 3.8 \times 10^7\) s\(^{-1}\) is the excited state linewidth, \(I\) is the laser intensity, and \(I_{\text{sat}} = 2.5\) mW/cm\(^2\) is the saturation intensity for linearly polarized light. For an
incident beam with power $P$ and Gaussian waist $w$, the intensity is given by

$$I(y) = \frac{8P}{\pi w^2} \cos^2(ky) = \text{const} + \frac{4P}{\pi w^2} \cos 2ky. \quad (3.12)$$

Once again, dropping the constant term and substituting (3.12) into (3.11) gives

$$\beta = \frac{\Gamma^2 P}{2\pi \Delta I_s w^2} \approx 10\hbar \omega_r \times \frac{P}{\text{mW}} \quad (3.13)$$

where the recoil frequency $\omega_r$ is $\hbar k^2/2M \approx 2.36 \times 10^4 \text{ s}^{-1}$. The best theoretical fidelity for the split operation occurs at $\beta = 2.8\omega_r$, or $P = 0.28 \text{ mW}$, about 20% different from the observed peak. We attribute this discrepancy to experimental errors in the intensity calibration. In particular, variations of this order can result from small misalignments of the beam on the atoms or deviations of the beam profile from a Gaussian. We therefore include a calibration factor in the theory curve, and adjust it to match the measured peak. The ratio between the powers for the splitting and reflection operations was about 0.65, in good agreement with theoretical expectation $\beta_{\text{reflect}} = 4.4\omega_r$.

**Higher-order split**

Reference [35] also points out that the double-pulse splitting sequence can be used efficiently for higher order operations, driving for instance $|0\rangle \to (|+2v_0\rangle + |-2v_0\rangle)/\sqrt{2}$. We implemented these operations as well. For the order-2 split above, we observed a peak fidelity of about 0.92, while the order-3 split produced atoms with velocity $\pm 3v_0$ at a fidelity of 0.8. Theory predicts fidelities of 0.99 and 0.97 respectively. For at least the third-order result, the relatively poor experimental performance can be attributed to the high sensitivity of the operation to the pulse intensity and duration. Our timing resolution is limited by the 100-ns switching time of the AOM, and we
observed the order-3 pulse to be sensitive on that time scale. We did not implement an order-4 split because it requires greater intensity than we had available.

**Reflect**

Similar results for the reflection operation are shown in Fig. 3.2. Here the fidelity is given by $N_{+2}$, since the initial velocity is close to $-v_0$. The optimal fidelity was about 0.94.

The optimum reflection fidelity is lower than that achievable for the splitting operation. It can be improved using a longer pulse with lower intensity [30], as shown in Fig. 3(b). The theoretical curve was calculated by varying the pulse length and at each point numerically optimizing the value of $\beta$. The reflection fidelity is observed to improve, though not monotonically. This can be understood with reference to the three-level model discussed previously. If the problem is solved in detail, it is found
that the initial state $|+v_0\rangle = (|+\rangle + |−\rangle)/\sqrt{2}$ evolves to

\[
|\psi\rangle = \frac{e^{-2i\omega_r t}}{\sqrt{2}} \left[ e^{-2i\omega_r t} |−\rangle - i \frac{\beta \sqrt{2} X}{X} \sin \frac{X t}{2} |0\rangle \right]
+ \left( \cos \frac{X t}{2} - i \frac{\omega_r X}{X} \sin \frac{X t}{2} \right) |+\rangle \] (3.14)

where $X = (2\beta^2 + 16\omega_r^2)^{1/2}$. The desired state $|-v_0\rangle = (|+\rangle - |−\rangle)/\sqrt{2}$ is achieved when $2\omega_r t = n\pi$ and $X t/2 = (n + 1)\pi$ for integer $n > 0$. The peaks observed in Fig. 3 indeed occur at $\omega_r t \approx n\pi/2$. Solving for the required intensity yields $\beta = 2\omega_r (4n + 2)^{1/2}/n$. The reflection operation is more accurate for large $n$ because coupling to higher velocity states is reduced as $\beta$ decreases.

We also experimentally observed the second maximum, at pulse time $t = 140 \mu s$ and $\beta = 3.0\omega_r$. The results are shown in Fig. 3.3, where a peak fidelity of about 0.98 is observed. The longer pulse duration, however, results in greater sensitivity to the
velocity. In situations where the velocity cannot be perfectly controlled, the shorter pulse may be preferable.

### 3.1.3 High fidelity pulses

For many applications, even higher-fidelity split and reflect operations will be required. These operations are especially advantageous if many are to be performed in order to get the atoms to follow some complex trajectory. An example of this is the use of these pulses to keep a condensate supported against gravity. Supporting ultra-cold atoms with high-fidelity pulses is an important part of this dissertation and will be described in detail in chapter 2, but this is one main motivation for the work that follows. The fidelities of the split and reflect pulses mentioned above are limited to about 0.99 and 0.94 respectively. Figure 3.4 shows an absorption image taken after reflecting the atoms and waiting 30 ms for atoms at different velocities to separate. It reveals the imperfectness of the pulse and how the pulse is exciting some atoms to undesirable momentum states.

**Pulse optimization**

Fortunately, we are not forced to accept such inferior pulses. By shaping the intensity-time profile, it is possible to construct a pulse that leaves fewer atoms in the wrong state. This problem of pulse shaping in general, does not have an exact solution as did the two level system described in Sec. 3.1.1. An exact solution has been shown for only some unique cases such as that of a pulse with a Gaussian time profile [41], however even the Gaussian shaped pulses do not give high fidelity.

To optimize our standingwave operations, we solve the Schrödinger equation in
Figure 3.4: Illustration of atoms that are left in undesirable momentum states after the application of our single-pulse reflect operation. Atoms were initially moving downward and then reflected up. After the pulse was applied, we waited 11 ms to allow packets with different velocities to separate. An absorption image was taken in the usual way by shining a resonant probe beam through the atoms and into a camera. The extra clouds in undesired momentum states represent about 6% of the total number of atoms.

the presence of an optical potential in much the same way as in we did in (??).

\[ i \frac{d\psi}{dt} = \left[ -\frac{\hbar}{2M} \frac{\partial^2 \psi}{\partial y^2} + \beta \cos(2ky)\psi \right], \quad (3.15) \]

We use the Bloch expansion

\[ \psi(y, t) = \sum_n c_n(t)e^{i(2nk+\kappa)y}, \quad (3.16) \]

where \( \kappa = M v_i / \hbar \) [??]. The coefficients \( c_n \) satisfy

\[ i \frac{dc_n}{dt} = \frac{\hbar}{2M} (2nk + \kappa)^2 c_n + \frac{\beta}{2} (c_{n-1} + c_{n+1}). \quad (3.17) \]

We solved equations (3.17) numerically, including indices \( n \) up to \( \pm 6 \). The theoretical
results obtained in previous sections using this method showed excellent agreement with experiment and show the reliability of the model. We apply a genetic algorithm to the model to optimized the pulse shape for the case of order-1, order-2, and order-4 splits and reflects.

Genetic algorithm

A genetic algorithm is a computational method for solving optimization problems using the process that drives evolution in biological systems, namely the method of natural selection. It works by repeatedly modifying a population of individual solutions. The genetic algorithm randomly selects individual solutions from the current population to be "parents" and uses them to produce the "children" of the next generation. The worst of the offspring are terminated while the best are allowed to breed. Over successive generations, the population "evolves" toward an optimal solution. The genetic algorithm can be applied to a variety of optimization problems that are not well suited for standard optimization algorithms, including discontinuous and highly nonlinear functions. It has previously been applied to the problem of manipulating cold atoms with an optical standing wave in [35] for the case of a double-pulse split operation.

We implemented a genetic algorithm for optimizing the population transfer from an arbitrary initial state to some desired final state using multiple standing-wave laser pulses to drive the transition. We used the MatLab programming language with the ga function to perform the optimization. A varying numbers of pulses were strung together and tested by the genetic algorithm. An example of such a pulse is illustrated in Fig. 3.5 showing a pulse train composed of three back-to-back pulses of varying intensities and times. For the sake of clarity, I will sometimes refer to these individual back-to-back pulses as steps. One could imagine setting up the experimental apparatus so that the genetic algorithm actually tested a real
Figure 3.5: An example of the intensity-time profile for a multiple-pulse standing-wave operation. This illustration is for a general pulse train composed of three back-to-back pulses with durations, $T_1, T_2,$ and $T_3$. The intensities for each section could similarly be labeled $\beta_1, \beta_2,$ and $\beta_3$.

pulse sequence on a real condensate and varied the next pulse based on the physical results. It should be emphasized that this was not the case. The genetic algorithm tested the pulses on the computer model and not with the actual experiment. The justification for this is several fold: (1) the cycle time for the experiment is about two minutes, while the computer can compute millions of solutions in the same time, (2) experimental detection accuracy is low compared to the precision of the computer, (3) the interfacing of the genetic algorithm to the experiment would have been expensive and time consuming, (4) the reliability of the computer model had already been thoroughly proven.

Split

Although the genetic algorithm was set up to handle solutions composed of many steps, we found that more than about 7 steps became significantly cumbersome to compute. Most good pulses seemed to follow the trend of being symmetric in time, so we constrained the algorithm to limit the search to pulses composed of three steps with the intensity of the first step equal to the intensity of third step.
Figure 3.6: Illustration of the three-pulse split operation. Atoms were initially at rest. After the pulse was applied, we waited 30 ms to allow packets with different velocities to separate. No atoms are visible in undesired momentum states.

<table>
<thead>
<tr>
<th>Order</th>
<th>$T_1$</th>
<th>$T_2$</th>
<th>$T_3$</th>
<th>$\beta_1$</th>
<th>$\beta_2$</th>
<th>$\beta_3$</th>
<th>1-Fidelity</th>
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</thead>
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<td>21.68</td>
<td>36.44</td>
<td>21.68</td>
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<td>43.11</td>
<td>31.42</td>
<td>43.11</td>
<td>11.85</td>
<td>1.78</td>
<td>11.85</td>
<td>$6.6 \times 10^{-4}$</td>
</tr>
</tbody>
</table>

Table 3.1: Triple-pulse orders 1 and 2 split parameters. Times, $T$, are given in $\mu s$ and intensities, $\beta$, are in units of $\omega_r$, where $\omega_r$ is the recoil frequency.

Some results for useful multi-pulsed split operations are given in table 3.1. The order gives the momentum transfer in units of $2\hbar k$ for example, order-$n$ split operation drives the $|0\rangle \rightarrow \frac{1}{\sqrt{2}}\left(|+2n\hbar k\rangle + | -2n\hbar k\rangle\right)$. They are all composed of three back-to-back pulses. It is interesting to note that the general shape of the split pulse was concave in the center with raised “wings” on the sides. Fig. 3.6 shows the physical realization of an order-1 tripple-pulse split operation. There are no atoms detected in the center of the two clouds.
Reflect

Our reflect pulses are each composed of three back-to-back pulses of varying times and intensities much like that of the split. The optimized times and intensities are given in table 3.2. Unlike the split, the shape of the reflects are peaked in the middle of the pulse while the “wings” of the pulse are usually about half as intense as the center. The numerical model predicts fidelities greater than 0.99 for each of these pulses. We implemented both the order-1 and order-2 reflects. When the pulses are peaked up, no atoms in undesired momentum states can be observed in the absorption images. This can be seen in fig. 3.7. A Gaussian fit to the background noise of the image gives less than 100 atoms giving us the ability to distinguish atom loss on the 1% level. It will be shown in the next chapter that through the repeated application of reflects we can verify that the fidelity of these pulses is better than 0.999. We did not have enough power in the laser to implement the order-4 reflect.
Figure 3.8: Comparison of reflect pulses. Shown are plots of pulse fidelity versus velocity in units of recoil velocity $v_r$. (a) compares single-pulse and triple-pulse order-1 reflects. (b) compares single-pulse and triple-pulse order-2 reflects. (c) shows order-4 reflect.

<table>
<thead>
<tr>
<th>Order</th>
<th>$T_1$</th>
<th>$T_2$</th>
<th>$T_3$</th>
<th>$\beta_1$</th>
<th>$\beta_2$</th>
<th>$\beta_3$</th>
<th>1-Fidelity</th>
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<td>61.7</td>
<td>10.8</td>
<td>2.28</td>
<td>4.59</td>
<td>2.28</td>
<td>$1.5 \times 10^{-4}$</td>
</tr>
<tr>
<td>4</td>
<td>33.46</td>
<td>69.4</td>
<td>33.46</td>
<td>6.81</td>
<td>14.59</td>
<td>6.81</td>
<td>$2.9 \times 10^{-3}$</td>
</tr>
</tbody>
</table>

Table 3.2: Triple-pulse reflect parameters for orders 1, 2, and 4 reflects. Times, $T$, are given in $\mu$s and intensities, $\beta$, are in units of $\omega_r$, where $\omega_r$ is the recoil frequency.

3.1.4 Summary

In summary, we have investigated the optical manipulation of atomic wave packets by an off-resonant standing wave laser. We demonstrated splitting and reflecting
with high fidelity and good agreement with theoretical expectations. We further improved fidelity by using a train of multiple pulses to drive the desired transitions and using a genetic algorithm to optimize the pulse shape. Through this method, we discovered many useful pulses using a three-pulse train, specifically order-1 (|0⟩ to ±|2ℏk⟩ and order-2 (|0⟩ to ±|4ℏk⟩) split operations as well as (±|ℏk⟩ → ±|ℏk⟩), (±|2ℏk⟩ → ±|2ℏk⟩), and (±|4ℏk⟩ → ±|4ℏk⟩) reflection operations. Numerical inaccuracies for these pulses were generally less than 1 part in 10^4 and experimentally we were able to verify that a single application of these pulses gave a fidelity near unity limited only by detection noise. These results will prove useful for the development of atom interferometers and other applications of atom optics.
In the previous chapter we developed and described the necessary machinery for manipulating the motion of ultra-cold atoms with high precision using a pulsed optical standing wave. The goal now is to reorient the standing wave vertically and investigate the possibility of keeping the atoms suspended against gravity by bouncing the atoms with our reflect pulses. Bouncing Bose-Einstein condensates has already been demonstrated by other groups. For example, in the work of Bongs et al., they bounce atoms from a blue-detuned light sheet [24] and in [23] Arnold et al. use a pulsed
magnetic mirror to bounce atoms. In each of these previous works, the number of consecutive bounces demonstrated has been less than three.

4.1 Motivation

We would like to bounce our condensate many more times than has previously been achieved using other methods. Doing so has significant implications for many proposed ideas and applications of cold atom physics. As an example, for some time people have wanted to study the dynamics of a Bose-Einstein condensate in a micro-gravity environment. Specifically, record low temperatures could be achieved through complete adiabatic expansion of a BEC. It is generally not possible to adiabatically decrease the trap confinement while in the presence of Earth’s gravity [8]. To date, several projects have been proposed to place a BEC apparatus on an orbiting platform, however funding for NASA’s Jet Propulsion Laboratories’ BEC project was recently cut and the project was scrapped. Another micro-gravity experiment is being performed in Germany by Ertmer et al. using a condensate apparatus in an evacuated drop tower [42]. A different project by Bouyer et al. at the Institut d’Optique in Orsay, France proposes to place an apparatus on an airplane that would then fly parabolic trajectories. Our method of bouncing atoms many times provides an artificial free-fall environment that could be useful and convenient for investigating the dynamics of a BEC in a micro-gravity environment, while the experiment and experimenters remain firmly planted on the Earth’s surface.

Perhaps the most obvious use of a bouncing condensate would be in a compact device for measuring the acceleration of gravity through ultra-cold atom interferometry. As has been detailed previously, hundreds, if not thousands of bounces will be required to be competitive with the sensitivity of current non-compact devices. Indeed, this is our main motivation for the development of the capability to perform
many bounces. In this chapter we report the use of optical standing-wave reflect operations to keep atoms suspended against gravity for more than 150 ms using 130 reflections.

4.2 Idea

When the trap is turned off, the atoms begin to fall due to gravity. The position and velocity are given as a function of time by

\[ x(t) = x_i + v_i t - \frac{1}{2} g t^2 \]  \hspace{1cm} (4.1)

and

\[ v(t) = v_i - gt \]  \hspace{1cm} (4.2)

where \( x_i \) is the initial position, \( v_i \) is the initial velocity, and \( g \) is the local acceleration from gravity at the surface of the Earth. Although \( g \) varies locally, it has an accepted average value of 9.801 m/s\(^2\). After some appropriate free-fall time, the atoms will be moving with a speed \(-nv_r\), where \( v_r = \frac{\hbar k}{M} = 5.88 \text{ mm} \) is the recoil velocity. A suitable optical standing wave is pulsed on to reflect the motion of the atoms by transferring momentum to the atoms in units of \(2\hbar k\). In this way, the pulse reflects downward-moving atoms at speed \(-nv_r\) to upward-moving with speed \(+nv_r\) where \( n \) is labeling the reflection order. A short time later the atoms will again be falling with a speed \(-nv_r\) and the process can be repeated.

Indeed, it would be possible to use our regular single-pulse reflect described in chapter 3 and in [43] to suspend the atoms. Unfortunately, the single-pulse reflect only puts 94% of the atoms into the desired momentum state and populates adjacent states with the remaining atoms. Imperfect reflect operations mean that the total number of suspended atoms will decay with each additional reflect pulse giving a
half-life of only 11 bounces. Furthermore, because the drop distance is less than the width of the condensate for some reflects, adjacent momentum states populated from one pulse overlap and can interfere with atoms during the following pulse resulting in even more atom loss. It is clear that pulses with much higher efficiencies are required to enable many bounces and so we utilize the high-fidelity three-step reflect pulses from chapter 3. Additionally, because the efficiencies of these pulses are dependent on both intensity and velocity, care must be taken when optimizing the pulse and its timing.

### 4.3 Setup

We use Bose-Einstein condensates consisting of about $10^4 \ ^{87}\text{Rb}$ atoms in the $F = 2$, $m_f = 2$ magnetically-trappable state as the source of ultra-cold atoms. The atoms are held in a weakly-confining magnetic TOP trap created by applying oscillating currents to our guide structure\cite{4}. For the experiments here the harmonic oscillation frequencies of the trap were $\omega_x = 2\pi \times 7.4 \ \text{Hz}$, and $\omega_y = 2\pi \times 0.8 \ \text{Hz}$, $\omega_z = 2\pi \times 4.3 \ \text{Hz}$. The BEC can be released from the trap by simply cutting the current to the trap to turn the fields off. We have two ways to control the trapping currents and each will be employed for different procedures described in this chapter. An analog channel allows us to set the amplitude of the currents to some arbitrary value by applying a voltage between -10 V (off) and 10 V (on), and a digital channel using TTL logic levels can be used to turn the current on or off to the prescribed analog level. The digital switch causes a $1/e$ turn-off time of 160 $\mu$s. The analog method of switching is much faster. The fields can be completely switched off in less than 100 $\mu$s by setting the analog signal to -10 V. The exact details of the switching circuit can be found in \cite{44}.

We manipulate the motion of the atoms using a pulsed optical standing wave. The
standing wave was produced by a homebuilt diode laser, the construction of which is outlined in Appendix B. The laser is tuned 27 GHz blue of the $5S_{1/2}$ to $5P_{3/2}$ laser cooling transition at a wavelength of 780.193 nm. A single acousto-optic modulator (AOM) was used to turn the beam on and off and a separate AOM was used to adjust the intensity. The output of the AOM was coupled into a single-mode fiber which provided spatial filtering and pointing stability. Two mirrors were used to direct the output beam from the fiber vertically through the vacuum cell. The beam from the fiber had a Gaussian intensity profile with a $1/e^2$ radius of about 1 mm. The fiber output coupler was mounted on a Newport 2-axis translating stage simplifying beam alignment to the atoms. An external mirror was mounted 20 cm above the vacuum chamber retro-reflecting the beam and generating the optical standing wave. The mirror was equipped with micrometers which provided a reference for the mirror’s orientation. When the retro mirror was properly aligned, the beam couples back in through the output fiber coupler and can be seen on a photodiode with a beamsplitter placed before the input fiber coupler. This method ensures that the beams are well-overlapped at the atoms.

The experiments here were performed by applying one or more reflect pulses and then allowing the atoms to evolve freely for 11 ms. During this time, packets with different velocities would separate in space. We observe the atoms by passing a resonant probe beam horizontally through the atoms and into a camera. The resulting images show the position of the clouds and the number of atoms can be calculated by fitting the image to a Gaussian profile. The imaging setup can be reconfigured quickly, through the use of kinematic mirror mounts, to get a different horizontal view of the atoms; one that differs by 45°. When the good three-step pules are used and the timings between pulses are properly set, only the main cloud is visible.
4.4 Results

The atoms are released from the trap by switching the analog control voltage to -10V. They leave the trap with some small initial velocity in the vertical direction due to the imperfectness of the trap turn-off. Sometime later, the order-1 reflect pulse is applied. The undesirable non-zero initial velocity of the condensate is compensated for by adjusting the timing between the trap release and the first reflect pulse. The initial drop time was 740 µs. After waiting a time $t = \frac{2\hbar k}{mg}$ from the start of the first reflect, the atoms will again be moving downward with the same velocity as they were before the first reflect. At this point we apply a second reflect. The process is cycled further for multiple bounces and an absorption image reveals the number of atoms remaining after the N-bounce cycle. The timing between pulses was first estimated using the accepted value of $g$. We then optimized the timing by progressively going to more bounces and adjusting the cycle time so that the atom number was a maximum. The optimized cycle time was later found to be 1.206 ms. Figure 4.1 shows a sequence of absorption images illustrating the remaining atoms after N order-1 bounces. We examine the efficiency of the reflect pulse by plotting the number of atoms remaining as a function of the number of applied reflect pulses. Fig. 4.2 shows that the order-1 reflect pulse is very good with fidelities indistinguishable from unity for the first 50 bounces. The pulses become inefficient after 50 bounces and the atom number decays sharply. We attribute this decay to several factors. The atoms are moving relative to the Gaussian beam profile. Additionally, the cloud is expanding. As the atoms move from the center of the beam the intensity at the atoms is reduced resulting in a decrease in pulse efficiency. Another factor could be the propagation of a timing error which would translate into a velocity after many bounces. Considering only the first 50 bounces and assuming an atom loss of about 1% (our detection limit), we infer a maximum pulse fidelity of 0.9998. This is in agreement with the numerical
Figure 4.1: Sequence of pictures showing atoms after N bounces with the 1st-order triple-pulse reflect. The number of bounces are indicated in each corresponding absorption image. The suspension of the cloud is illustrated above by the image labeled “0” which was taken by running the one-bounce sequence with the beam forming the standing wave blocked so that no reflect operation was performed. The timing between pulses was optimized by progressively going to more bounces and then adjusting the timing so that the atom number was a maximum. For more than 100 bounces, timing steps of 1µs had a noticeable effect on the number of remaining atoms. Also interesting to note is the fact that the atoms are being accelerated to the right due to misalignment of the standing wave.
Figure 4.2: Bouncing with order-1 reflects. Multiple order-1 reflect pulses are applied to keep a falling condensate supported against gravity. Plotted is the number of remaining atoms as a function of the number of applied reflect pulses. The atom number stays essentially constant for the first 50 bounces and then falls off sharply. We attribute this fall-off to the fact that the atoms are moving out of the beam.

In a similar fashion as above, we suspended atoms by implementing our order-2 reflect. The initial drop time was 1.35 ms and the cycle time was twice that of the order-1 bouncing. In figure 4.3 the number of remaining atoms is plotted after $N$ bounces with the order-2 reflect operation. With further optimization, we could still see atoms after more than 130 order-1 bounces and after 50 order-2 bounces. However, the maximum level of performance varied spuriously from day to day for large numbers of bounces.
Figure 4.3: Multiple order-2 reflect pulses are applied to keep the atoms supported against gravity. Similar to the order-1 bouncing, here the atom number stays constant for 20 bounces and then decays rapidly.

### 4.4.1 Beam alignment

It is apparent in the part of Fig. 4.1 showing 20, 50, and 100 bounces that the cloud is being accelerated horizontally. This indicates that the beam is not exactly aligned with gravity. Traditionally, getting a laser beam exactly vertical is quite challenging and for gravimetry experiments, it is exceptionally critical[45, 6]. Observing the horizontal motion after many bounces makes for an excellent tool to facilitate beam alignment. By turning the micrometers on the retro mirror the angle of the standing wave can be precisely adjusted with a resolution of about 70 µrad. We used the micrometers to make small adjustments to the angle of the beam and we used one of the mirrors before the vacuum chamber to peak up the light that was coupled back
into the fiber, ensuring that the upward and downward traveling beams were well overlapped. In this fashion, we were able to reduce the horizontal displacement of the cloud to less than 20 $\mu$m after 50 order-1 bounces.

### 4.4.2 Gravity

In the spirit of Galileo’s experiments of dropping masses from the Leaning Tower in Pisa, we could make a crude measurement of $g$ by simply dropping our atoms and timing their fall. Equivalently, we could get a value for $g$ by repeatedly imparting a known upward velocity to the atoms and adjusting the time between kicks required to keep the atoms suspended. Previously, we utilized the velocity dependence of the reflect pulse to ensure that the timing between pulses was correct. This was done by progressively applying more bounces, while making small adjustments to the cycle time in order to maximize the number of remaining atoms. For the order-1 bouncing, the optimum cycle time of 1.206ms was determined. Changes of less than 1 $\mu$s were resolvable by giving a noticeable change in the number of atoms remaining. The acceleration of gravity is given by

$$g = \frac{2\hbar k}{mt_0}$$

where $k = 2\pi/\lambda$ is the wave number of the laser, $m$ is the mass of $^{87}$Rb, and $t_0$ is the cycle time. The uncertainty, $\delta g$, is given by

$$\delta g = \frac{dg}{dt} \delta t$$

where $\delta t$ is the minimum resolvable time step. Plugging in physical constants and experimental parameters gives $g = 9.759 \pm 0.008\text{m/s}^2$.

Immediately noticeable is that our measured value differs significantly from the
accepted value for \( g \) of 9.81m/s\(^2\). Our method of measuring the acceleration does not distinguish the between the types of forces acting on the atoms. The discrepancy can be accounted for by considering the gradient of the magnetic field in the region of the atoms. The atoms are still in the \(|F = 2, m_F = 2\rangle\) state and so they will experience a force in the presence of a magnetic field gradient. Although it was difficult to measure the field gradient at the location of the atoms, it was easily estimated by measuring the difference in the field above and below the vacuum chamber and dividing by the distance between the measurement points. The value that we measured was \( B_z' = 0.07 \pm 0.02 \) and the force is given by

\[
F_z = \mu_z B_z' = \mu_B g_F m_F B_z'
\] (4.5)

where \( \mu_B \) is the Bohr magneton, \( g_F \) the Landé \( g \)-factor, \( m_F \) the magnetic sublevel. The acceleration that the atom experiences due to the field gradient is found by dividing the force, \( F_z \) by the mass of \( ^{87}\text{Rb} \). This gives a correction of 0.045\( \pm \)0.013m/s\(^2\) to our measured value of \( g \). The new value is 9.804 \( \pm \)0.015m/s\(^2\), which agrees with the accepted value within measurement uncertainty. A more accurate measurement of \( g \) could be obtained by using atoms in the \( m_f = 0 \) state or by directly measuring the gradient at the location of the atoms.

### 4.4.3 Stern-Gerlach

The Stern-Gerlach experiment, performed first in 1922, is named for Otto Stern and Walther Gerlach and was significant in showing the quantized nature of the magnetic moment of particles. The traditional experiment is illustrated in figure 4.4 and works by passing a beam of particles through a region of high magnetic field gradient. The particles, upon passing through the region of high field gradient, experience the deflecting force described in equation 4.5. The amount of deflection depends on the
spin of the particle. If the field is well-characterized, the magnetic properties of the particles can be determined by examining the amount of deflection. Alternatively, the Stern-Gerlach technique can be used to deduce the field gradient if the magnetic moments of the particles are already known. One such example of the utility of this technique is the measurement of the magnetic properties of Chromium clusters by Payne et al. [46] using a 0.25 m long gradient magnet at 360 T/m and interaction times $\sim 100 \mu s$.

Here we implement a Stern-Gerlach experiment with our bouncing technique using ultra-cold atoms localized better than 50 $\mu m$ to the field, with $\mu T/m$ gradients and interaction times of 0.1s. This scheme represents a completely different regime than has previously been demonstrated and illustrates one of the many possible uses of stationary, free-falling atoms. Furthermore, with this method we will directly measure the gradient at the location of the atoms.

We begin by mixing the m-levels of the atoms in the condensate by adjusting our trap turn-off procedure. Using the digital channel to turn off the trap causes the zero of the magnetic field to pass through the condensate and excites Majorana spin-flip
transitions. Under normal operating conditions, these spin-flips cause an undesirable loss of atoms from the trap. In this case, however, the spin flips occur as the trap is turned off and the atoms do not have time to leave.

The usual bouncing experiment was then performed using order-1 reflect pulses. No gradient was applied other than that of the ambient fields that exist in the lab. The cloud seemed to elongate after only a moderate number of bounces and after many bounces (as few as 30) we can distinguish two or three separate clouds. Figure 4.5 shows three distinct clouds of atoms each representing a different spin state. The gradient is determined by noting the distance between two clouds, say the $m_F = 0$ and the $m_F = 2$ clouds, and calculating the relative acceleration of these two clouds.
The relative acceleration is given by

\[
\Delta \vec{a} = \vec{a}_2 - \vec{a}_0 = 2(\vec{x}_2 - \vec{x}_0)/T^2
\]  

(4.6)

where \( x \) is the position vector of the respective cloud and \( T \) is the total bouncing time. The gradient is then given by

\[
\vec{B}' = \frac{m \Delta \vec{a}}{\mu_B g_F \Delta m_F}.
\]  

(4.7)

Using the values for the clouds in figure 4.5 we calculate the components of the acceleration \( a_x = 0.0369\pm 0.0005 \text{ m/s}^2 \) and \( a_y = 0.0550\pm 0.0005 \text{ m/s}^2 \) for atoms in the \( m_F = 2 \) due to the field gradient. Our corrected value of \( g \) is then \( 9.814\pm 0.008 \text{ m/s}^2 \). This corresponding components of the field gradient are \( B'_x = 287\pm 4 \mu \text{T/m} \) and \( B'_y = 428\pm 4 \mu \text{T/m} \).

4.5 Summary

We have demonstrated that high-fidelity reflect pulses generated by a pulse-shaped optical standing wave can be used to keep atoms suspended against gravity. Atoms were still present after more than 130 order-1 reflects; more than 40 times the number of bounces achievable by the previous methods of other groups. A total “hang time” of 0.15 s for the order-1 reflect was achieved. We were able to perform more than 50 order-2 reflects for “hang times” approaching 0.12 s. The optical standing wave was aligned vertically by monitoring the horizontal displacement of the cloud after many bounces and a value of \( g = 9.814\pm 0.008 \text{ m/s}^2 \) was determined by optimizing the time between bounces. Additionally, using our bouncing technique to sample a region of only 50 \( \mu \text{m} \), the Stern-Gerlach effect from ambient field gradients was
observed and measured to an accuracy of $4 \mu T/m$. This illustrated the possible use of bouncing condensates as a compact sensor for magnetic field gradients. We are now ready to move forward, applying these results to a vertically-oriented condensate interferometer.
In the previous chapter, we demonstrated the ability to precisely suspend free-falling atoms for long times by applying many bounces with an off-resonant optical standing wave. The natural progression and indeed, the underlying motivation is to now apply what we’ve learned to set up a vertically oriented interferometer with ultra-cold atoms. Atom interferometry has already proven itself unrivaled in sensitivity for many types of measurements, among which are gravitational and inertial sensing. Typically, devices used for measuring gravity that do not use ultra-cold atoms utilize
an atomic fountain geometry where atoms are captured in a Magneto-optical trap, laser cooled to \( \sim 1.5 \) \( \mu \)K, and launched vertically. During free-fall, Raman transitions between internal states are driven with vertically oriented laser beams to manipulate the motion of the atomic wavepackets. In this way the free-falling wavepacket is split vertically in two, reflected, and then recombined. The top packet gains phase relative to the bottom packet, and the measured phase can be related to the local value of \( g \). Large packet separations (\( \sim 1 \) mm) and long interaction times, which equate to high sensitivities, are achieved because of large drop distances (\( \sim 1 \) m).

### 5.1 Motivation

The value of Earth’s gravity depends strongly on where you are; both your elevation and latitude. To a high enough precision, it even depends on what’s near you. A very sensitive gravity-measuring device could measure small local deviations in the acceleration of gravity due to the non-uniform mass distribution of the Earth, enabling a myriad of useful applications. For instance, gravitational mapping could allow vehicles to navigate and function autonomously. Satellite trajectories could be calculated with higher precision. Ocean levels, polar ice, and tectonic activity could be monitored more accurately. Geologists could more easily discover underground mineral and oil deposits. And the Pentagon could finally detect certain caves and enemy bunkers. The possible applications are many and the prospects for gravimetry using cold atoms are at the forefront and quickly maturing.

Free-falling atoms are a prerequisite for most proposed gravimetry devices using atom-interferometric methods. Because the sensitivity of these devices increases with larger packet separation and longer separation time, the simplest way to achieve higher sensitivity is by making larger devices and allowing the atoms to drop farther. Currently a project is underway by Kasevich \textit{et al.} to build an apparatus that would
allow for a 10 m drop distance [47] with fall times nearing 2 seconds and packet separations of several centimeters.

Although larger devices will generally mean increased sensitivity, for some applications requiring portability, instruments even 1 m in length are much too large. In chapter 4 we demonstrated one method of measuring gravity by essentially dropping ultra-cold atoms and timing their fall. Due to repeated bouncing, our method required a negligible vertical drop distance of \( \sim 10 \mu \text{m} \). In this chapter we show that our previously developed techniques can be used to create a vertically oriented interferometer that is sensitive to gravity and also requires a negligible drop distance. We implemented two different configurations of vertical interferometers where the packet separations were greater than the drop distance. This is in stark contrast to the fountain devices where the drop distance increases as the square of the packet separation. In other work, Ferrari et al. measure the acceleration of gravity by observing Bloch oscillations of cold atoms in a periodic trapping potential. Their work is notable for being compact and in later discussion it will be directly compared to our method. Our general method will be described here and a more detailed discussion of each specific variant will follow.

The atomic wavepacket, upon being released from the trap, is split using similar pulses described in chapter ?? . A superposition of upward-moving and downward-moving atoms is created. Additional pulses from the optical standing wave are used to apply reflects operations at precise times, keeping the atoms suspended against gravity. The timing between reflects is chosen so that the trajectory of the initially upward-moving packet is always above the trajectory of the initially downward-moving packet. Care must be taken so that each reflect pulse affects both packets in the desired way. With each successive cycle additional phase is acquired. The packets are recombined after the desired number of cycles and the phase is measured.
5.2 Configuration 1

This configuration, although performed last chronologically, will be presented first because of its simple implementation and impressive results. As will be seen, the other configuration have a more complicated optical setup, requiring a two-color traveling wave while this one uses essentially the exact same optical setup as was used in the bouncing experiments, namely a single-frequency beam, vertically oriented and retro-reflected to produce a standing wave. It will be clear later how the similarity of this optical setup to that used for the bouncing experiments will allow us to more directly compare the measured values of gravity. With configuration 1 we were able to apply 81 successive operations to achieve 40 cycles for a total interferometer time of nearly 50 ms. The number of operations performed here has only been exceeded by our own bouncing experiments described previously and indeed, never before has so many individual pulses been used in an atom interferometer of any kind.

5.2.1 Experiment

The optical setup is essentially identical to that described in the bouncing experiments. An optical standing wave is used to manipulate the motion of condensates consisting of about $10^4 \ ^{87}\text{Rb}$ atoms. The standing wave is formed by an external cavity stabilized diode laser tuned 27 GHz blue of the $5S_{1/2}$ to $5P_{3/2}$ laser cooling transition at a wavelength of 780.193 nm. One AOM is used to turn the beam on and off while a second AOM is used to set the intensity. The output of the AOM is coupled into a single-mode fiber and routed to the experiment. The output of the fiber is retro-reflected by a mirror located 20 cm above the vacuum chamber to create the standing wave.

The trajectories for the atoms in configuration 1 are depicted in figure 5.1. The trap currents are turned off in less than 100 $\mu$s and the atoms begin to fall under
Figure 5.1: Schematic illustrating the trajectory of wavepackets in config. 1. The atoms are released from the trap and begin to fall. When the atoms are moving at $-v_r$, a $\pi/2$ operation is performed by a single 34.2 $\mu$s pulse of the standing wave with intensity $\beta = 1.95\omega_r$ forming the superposition $|\psi\rangle = \frac{1}{\sqrt{2}}(|+v_rangle + |-v_r\rangle)$. The motion of the packets is made cyclical by alternating order-2 reflects and order-1 reflects every time interval, $t_1$. The order-1 reflect is represented by a skinny arrow, the order-2 by a wide arrow, and the split and recombination by a tall arrow.

the force of gravity. After an appropriate time the atoms are moving downward with a velocity $-v_r$ and we split the cloud with a $\pi/2$ operation. The $\pi/2$ operation is performed by a single 34.2 $\mu$s pulse of the standing wave, driving the transition $|-v_r\rangle \rightarrow \frac{1}{\sqrt{2}}(|+v_r\rangle + |-v_r\rangle)$. The intensity of the standing wave for this operation is $\beta = 1.95\omega_r$ where $\omega_r$ is the recoil frequency. After the split pulse, the top packet is decelerated by gravity while the bottom packet is accelerated downward. A time, $t_1 \approx \frac{\omega_r}{g}$ later, the top packet is nearly stationary while the bottom packet is moving at $-2v_r$. At this point, we apply our order-2 three-pulse reflect which reverses the motion of the bottom packet and leaves the trajectory of the top packet unchanged.

The same time interval later, the top packet is now moving down at $-v_r$ while the bottom packet is moving up at $+v_r$. We apply our order-1 three-pulse reflect reversing
the motion of each packet. In this way the initially upward-moving packet continues
to be above the initially downward moving packet. The cycle is then repeated a
desired number of times by alternating the application of order-2 and order-1 reflect
operations every $\Delta t \approx \frac{v_r}{g}$. After $N$ cycles the packets are recombined by applying
another $\pi/2$ operation. Note that the $\pi/2$ pulse replaces what would have been
another order-1 reflect had the cycling continued.

We use the times determined from the order-1 bouncing experiment to get the
timing correct for the vertical interferometer. The time between trap turn-off and
application of the split pulse is 740 $\mu$s. The time interval between all interferometer
pulses is 603 $\mu$s and times are accurate to better than 1 $\mu$s.

An absorption image is taken following an 11 ms time-of-flight after the $\pi/2$
recombination pulse. This short delay allows the packets in the final interferometer
output to separate in space. We obtain the number of atoms in each output port by
fitting the clouds in the absorption image using a two-dimensional Gaussian function.

The accumulated phase is related to the fraction of atoms by

$$\frac{N_0}{N} = \cos^2(\phi/2 - \alpha)$$

where $N_0$ is the number of atoms in the bottom cloud after the time-of-flight, $N$
is the total number of atoms in both clouds, $\phi$ is the differential phase acquired by
the packets due to gravity, and $\alpha$ is a known phase applied to the interferometer by
shifting the phase of the standing wave before the final recombination.

In the experiment we can observe the operation of the interferometer by applying
a known phase $\alpha$ to change $N_0/N$, the fraction of atoms that end up in the bottom
cloud after recombination. The phase of a standing wave is given by $\phi_{sw} = 2k_lD$
where $k_l$ is the laser wave number and $D$ is the distance from the atoms to the retro-
mirror. A change $\alpha = \Delta \phi_{sm}$ could then be applied to the interferometer by changing
either the position of the mirror or the laser wave number during the interferometer sequence. Since the position of the mirror is fixed at $D \approx 20$ cm, $\alpha$ is most easily adjusted by adjusting the laser wave number by changing the frequency of the laser producing the standing wave before the final recombination. The applied phase is then expressed in terms of the frequency shift $\Delta \nu$ by

$$\alpha = \frac{4\pi D}{c} \Delta \nu$$  \hspace{1cm} (5.2)

The frequency of the laser can be changed in less than 10 $\mu$s by modulating the current to the diode. Total current modulation is $\pm 4$ mA corresponding to a frequency shift $\sim \pm 1$ Ghz. Plotting the ratio $N_0/N$ as a function of the applied phase (or rather the control voltage to the diode current modulation) gives an interference curve. Interferometer curves for 1, 4, and 40 cycles are shown in figure 5.2.

Each curve is fit to the functional form

$$f(x) = y_0 + A \cos \left( \frac{2\pi x}{P} + \phi \right)$$  \hspace{1cm} (5.3)

where $y_0$ is the average value and expected to be close to 0.5, $A$ is the amplitude, $P$ is the period, and $\phi$ is the phase. The visibility $V$ is another important parameter that can be extracted from this fit.

The visibility is related to the contrast of an interference pattern and is therefore a good measure of the quality of our interferometer. For light, the visibility of an interference pattern is given by

$$V = \frac{I_{\text{max}} - I_{\text{min}}}{I_{\text{max}} + I_{\text{min}}}$$  \hspace{1cm} (5.4)

where $I_{\text{max}}$ and $I_{\text{min}}$ is the maximum and minimum intensities of the fringe. For an
Figure 5.2: Interferometer curves for 1, 4, and 40 cycles of configuration 1.
atom interferometer the visibility is given by

\[ V = \left( \frac{N_0}{N} \right)_{\text{max}} - \left( \frac{N_0}{N} \right)_{\text{min}} \]  \hspace{1cm} (5.5)

or equivalently, from the fit parameters

\[ V = \frac{A}{y_0}. \]  \hspace{1cm} (5.6)

### 5.2.2 Quantum mechanical phase

Before discussing the experimental results, we take the opportunity to diverge slightly from the physical experiment to focus on the quantum mechanics of the system. Here we develop the formalisms required to understand how the quantum phase develops during the interferometer sequence. Although, this is done in reference to the trajectories of configuration 1, much of what follows will be relevant to the other configuration as well.

**Propagator**

The first thing to work out is the quantum evolution of a particle falling under the influence of gravity. The Schrödinger equation for such a particle can be written

\[ i\hbar \frac{\partial \Psi}{\partial t} = \left( -\frac{\hbar^2}{2m} \nabla^2 + mgz \right) \Psi \]  \hspace{1cm} (5.7)

Take the initial state to be \( \Psi(z,0) = e^{ikz} \) for arbitrary \( k \) and try a general solution of the form

\[ \Psi(z,t) = e^{i(k - \frac{mgz}{\pi})z}e^{i\phi(t)}. \]  \hspace{1cm} (5.8)
Taking the partial derivative with time and the Laplacian of Eq. (5.8) gives

\[ \frac{\partial \Psi}{\partial t} = -i\left( \frac{mgz}{\hbar} - \frac{\partial \phi}{\partial t} \right) \Psi \]

(5.9)

and

\[ \nabla^2 \Psi = -(k^2 - \frac{2kmgt}{\hbar} + \frac{m^2g^2t^2}{\hbar^2}) \Psi. \]

(5.10)

Substituting Eqs. (5.9) and (5.10) back into (5.7) gives the differential equation

\[ \frac{d\phi}{dt} = -\frac{\hbar k^2}{2m} + gkt - \frac{mg^2t^2}{2\hbar} \]

(5.11)

Integrating gives the phase acquired for a particle falling for time \( t \)

\[ \phi(t) = -\frac{\hbar}{2m} \left[ k^2 t - k\gamma t^2 + \frac{\gamma^2 t^3}{3} \right]. \]

(5.12)

where \( \gamma = \frac{mg}{\hbar} \) has been substituted for simplicity. Notice the \( t^3 \) term does not depend on \( k \) or \( z \), so it will contribute some overall phase. In interferometry, a common phase to both arms is not relevant and can be dropped. Our simplified expression for the phase gained by a free-falling particle in our interferometer is then

\[ \phi(t) = -\frac{\hbar}{2m} [k^2 t - k\gamma t^2]. \]

(5.13)

We can now propagate \( \Psi(z, 0) = e^{ikz} \) for fall time \( t \) obtaining

\[ \Psi(z, t) = e^{i(k-\gamma)z} e^{-i\frac{\hbar}{2m} [k^2 t - k\gamma t^2]}. \]

(5.14)
Single cycle

Having worked out the quantum phase evolution for a free-falling atom, we are now ready to analyze a single cycle of our interferometer. During this treatment we specifically leave out the phase acquired by from the reflect pulses. We begin with the initial state of our falling atoms

\[ \psi = e^{-i(-k_0 + \delta)z} \]  \hspace{1cm} (5.15)

where \( \hbar \delta/m \) is to account for any error in the initial velocity and \( k_0 = k_L = 2\pi/\lambda \). We split the cloud with our \( \pi/2 \) pulse and get

\[ \psi \rightarrow \frac{1}{\sqrt{2}}[e^{-i(-k_0 + \delta)z} - ie^{-i(k_0 + \delta)z}] \]  \hspace{1cm} (5.16)

We now propagate Eq. (5.16) for time \( t \) and get

\[ \psi \rightarrow \frac{1}{\sqrt{2}} \left[ e^{-i(-k_0 - \gamma t + \delta)z}e^{-i \frac{\hbar}{2m}[(k_0 + \delta)^2 t - (k_0 + \delta)^2 \gamma t^2]} - e^{-i(k_0 + \gamma t + \delta)z}e^{-i \frac{\hbar}{2m}[(k_0 + \delta)^2 t - (k_0 + \delta)^2 \gamma t^2]} \right] \]  \hspace{1cm} (5.17)

\[ = \frac{1}{\sqrt{2}} \left[ e^{-i(-k_0 - \gamma t + \delta)z}e^{-i\Theta_1} - ie^{-i(k_0 - \gamma t + \delta)z}e^{-i\Theta_2} \right] \]

where \( \Theta_1 \) and \( \Theta_2 \) have been introduced to simplify the expression. Next we apply the order-2 reflect pulse when \( k_0 + \gamma t \approx 2k_0 \) giving

\[ \psi \rightarrow \frac{1}{\sqrt{2}} \left[ e^{-i(3k_0 - \gamma t + \delta)z}e^{-i\Theta_1} - ie^{-i(k_0 - \gamma t + \delta)z}e^{-i\Theta_2} \right] \]  \hspace{1cm} (5.18)
We propagate for another time \( t \) giving
\[
\psi \rightarrow \frac{1}{\sqrt{2}} \left[ e^{-i(3k_0 - 2\gamma t + \delta)z} e^{-i\Theta_1} e^{-i\frac{\hbar}{2m}(3k_0 - \gamma t + \delta)^2 t - (3k_0 - 2\gamma t + \delta)\gamma t^2} \\
- ie^{-i(k_0 - 2\gamma t + \delta)z} e^{-i\Theta_2} e^{-i\frac{\hbar}{2m}(k_0 - \gamma t + \delta)^2 t - (k_0 - \gamma t + \delta)\gamma t^2} \right]
\]
(5.19)
\[
= \frac{1}{\sqrt{2}} \left[ e^{-i(3k_0 - 2\gamma t + \delta)z} e^{i(-\Theta_1 - \Theta_3)} - ie^{-i(k_0 - 2\gamma t + \delta)z} e^{i(\Theta_2 - \Theta_4)} \right]
\]
where \( \Theta_3 \) and \( \Theta_4 \) have been substituted. After a single cycle, recombination of the two packets gives
\[
\psi_{final} = \frac{1}{2} \left[ e^{i(3k_0 - 2\gamma t + \delta)z} \left( e^{i(-\Theta_1 - \Theta_3)} - e^{i(-\Theta_2 - \Theta_4)} \right) \\
- ie^{i(k_0 - 2\gamma t + \delta)z} \left( e^{i(-\Theta_1 - \Theta_3)} + e^{i(-\Theta_2 - \Theta_4)} \right) \right]
\]
(5.20)
and the fraction of atoms in each port is given by
\[
N_{+h} = \frac{1}{4} \left| e^{-i(\Theta_1 + \Theta_3)} - e^{-i(\Theta_2 + \Theta_4)} \right|^2 = \sin^2\frac{\Theta}{2}
\]
(5.21)
\[
N_{-h} = \frac{1}{4} \left| e^{-i(\Theta_1 + \Theta_3)} + e^{-i(\Theta_2 + \Theta_4)} \right|^2 = \cos^2\frac{\Theta}{2}
\]
where \( \Theta = \Theta_1 + \Theta_3 - \Theta_2 - \Theta_4 \). Substituting back in for \( \Theta_1, \Theta_2, \Theta_3, \) and \( \Theta_4 \) from above gives the total phase from one bounce
\[
\Theta = \frac{2\hbar}{m} (2k_0^2 t - k_0 \gamma t^2) = 2v_0 t(2k_0 - \gamma t).
\]
(5.22)
Setting $t = \frac{n_\text{e}}{g}$ gives the phase that is acquired by the interferometer in one cycle.

$$\Theta = \frac{2mv_0^3}{h g} = -18\pi \approx -55 \text{ rad} \tag{5.23}$$

**Multiple cycles**

We can similarly work out the acquired phase for multiple interferometer cycles. This time we will keep track of the Stark phase gained by the order-2 reflect pulse, while still neglecting the phase of the order-1 reflect because it is common to both packets.

We begin just after the application of the reflect at the beginning of the $n^{th}$ cycle and work out the phase accumulated through the beginning of the $(n + 1)^{th}$ cycle. The initial state is then given by

$$\psi_n = \frac{1}{\sqrt{2}} \left[ e^{i(k_0 + \delta_n)z} + e^{i(-k_0 + \delta_n)z} e^{i\phi_n} \right] \tag{5.24}$$

where $\phi_n$ is the phase of the standing wave during the previous operation. We propagate for time $t$ and get

$$\psi_n \rightarrow \frac{1}{\sqrt{2}} \left[ e^{i(k_0 - \gamma t + \delta)z} e^{i\Theta(k_0 + \delta_n)} + e^{i(-k_0 - \gamma t + \delta_n)z} e^{i\phi_n} e^{i\Theta(-k_0 + \delta_n)} \right] \tag{5.25}$$

where

$$\Theta(k) = \frac{\hbar}{2m} \left[ -k^2 t + \gamma k \dot{t}^2 \right]. \tag{5.26}$$

After the order-2 reflect we get

$$\psi_n \rightarrow \frac{1}{\sqrt{2}} \left[ e^{i(k_0 - \gamma t + \delta_n)z} e^{i\Theta(k_0 + \delta_n)} + e^{i(3k_0 - \gamma t + \delta_n)z} e^{i(\phi_n + \phi_r) + i\Theta(-k_0 + \delta_n)} \right] \tag{5.27}$$
where $\phi_{r2}$ is the phase acquired from the order-2 reflect. We propagate for another time $t$ and get

$$
\psi_n \to \frac{1}{\sqrt{2}} \left[ e^{i(k_0 - 2\gamma t + \delta_n)z} e^{i\Theta(k_0 + \delta_n)} e^{i\Theta(k_0 - \gamma t + \delta_n)} 
+ e^{i(k_0 - 2\gamma t + \delta_n)z} e^{i\Theta(-k_0 + \delta_n)} e^{i\Theta(3k_0 - \gamma t + \delta_n)} \right].
$$

(5.28)

Now applying an order-1 reflect gives

$$
\psi_n \to \frac{1}{\sqrt{2}} \left[ e^{i(3k_0 - 2\gamma t + \delta_n)z} e^{i\Theta(k_0 + \delta_n)} e^{i\Theta(k_0 - \gamma t + \delta_n)} 
+ e^{i(k_0 - 2\gamma t + \delta_n)z} e^{i\Theta(-k_0 + \delta_n)} e^{i\Theta(3k_0 - \gamma t + \delta_n)} \right].
$$

(5.29)

Factoring out the phase terms gives the accumulated phase through the beginning of the $(n + 1)^{th}$ cycle as

$$
\phi_{n+1} = \phi_n + \phi_{r2} + \phi_{r1} + \Theta(-k_0 + \delta_n) + \Theta(3k_0 - \gamma t + \delta_n) - \Theta(k_0 + \delta_n) + \Theta(k_0 - \gamma t + \delta_n).
$$

(5.30)

Substituting in for $\Theta$ using equation (5.26) gives

$$
\phi_{n+1} = \phi_n + \phi_{r2} + \frac{\hbar k_0}{m} \left[ - 4k_0 t + 2\gamma t^2 \right].
$$

(5.31)

For the total phase from an interferometer consisting of $N$ cycles we must include the phase of the initial split $\phi_0$ and the phase of the recombination $\phi_{rec}$. The phase for the total sequence consisting of $N$ cycles is given by

$$
\phi_{total} = N \left( \phi_{r2} + \frac{\hbar k_0}{m} \left[ - 4k_0 t + 2\gamma t^2 \right] \right) + \phi_0 + \phi_{rec}.
$$

(5.32)
A value for $\phi_{r_2}$ can be determined by solving the Schrödinger equation in the presence of the optical field of each respective reflect pulse. We could use our numerical model first introduced in section 3.1.2 to calculate the phase due to application of the optical lattice. This however would not account for the phase due to the action of gravity during the pulse. For this we turn to the interaction picture.

### 5.2.3 Interaction picture

The idea of pictures in quantum mechanics deals with the nature of states and operators and how the to interpret the time dependence of a problem. In the Schrödinger picture the time dependence is contained in the state vector and the operator is static. In the Heisenberg picture the state is static and the operators evolve in time. The interaction picture is considered “intermediate” between these other two pictures. In the interaction picture, both the state vector and the operators evolve in time, however, the time-evolution is determined by a perturbation. We make use of the interaction picture to include the effect of gravity on the phase in the presence of the optical standing wave.

We use the “falling” states from Eq. (5.14) as a basis and write the total Hamiltonian as

$$H = H_0 + H'$$

where $H_0 = \frac{p^2}{2m} + mgz$ is the unperturbed Hamiltonian and the standing wave is the perturbation $H' = \hbar \beta \cos[2k_0 z + \phi]$. In the interaction picture the state is is expressed

$$|\psi(t)\rangle = e^{iH_0 t/\hbar} |\psi(t)\rangle = U_0 (-t) |\psi(t)\rangle$$

where $U_0$ is the time evolution operator for $H_0$. The effect of $U_0$ on a state of momentum $p$ was determined from equation (5.12) and can be expressed in slightly different
terms as

$$U_0 |p⟩ = |p - mgt⟩ \times e^{-\frac{i}{\hbar} \left[ \frac{p^2}{2m}t - \frac{pg^2}{2}t^2 + \frac{1}{6}mg^2t^3 \right]}.$$  (5.35)

In the interaction picture, the Schrödinger equation becomes

$$i\hbar \frac{d}{dt} |\psi⟩_I = H'_I |\psi⟩_I$$  (5.36)

where

$$H'_I = e^{iH_0 t/\hbar} H' e^{-iH_0 t/\hbar}.$$  (5.37)

We use the basis states

$$|κ_n⟩ = |hk(2n + δ)⟩$$  (5.38)

which are stationary in the interaction picture. Here δ is the momentum offset at $t = 0$. $H'_I$ can be expressed as a matrix with elements given by

$$\langle κ_n | H'_I | κ_m⟩ = \langle κ_n | U^+_0(t)H'U_0(t) | κ_m⟩ = e^{i(Θ_n - Θ_m)} \langle hk(2n + δ) - mgt | H' | hk(2n + δ) - mgt⟩$$  (5.39)

where

$$Θ_n = -\frac{1}{\hbar} \left[ \frac{\hbar^2 k^2}{2m} (2n + δ)^2 t - \frac{hkg}{2} (2n + δ)t^2 + \frac{1}{6} mg^2 t^3 \right]$$  (5.40)

Starting with the matrix elements for

$$\langle k_n | H' | k_m⟩ = \frac{\hbar β}{2} \left( e^{iφ} δ_{n=m+1} + e^{-iφ} δ_{n=m-1} \right)$$  (5.41)

we see that $n = m \pm 1$. We now write

$$\langle κ_{n±1} | H'_I | κ_n⟩ = \frac{\hbar β}{2} e^{iφ} e^{i(Θ_n - Θ_{n±1})} = \frac{\hbar β}{2} e^{i[φ + 4ωt(2n±(1+δ)) - kgt^2]}$$  (5.42)
Using the expansion $|\psi\rangle = \sum_n c_n |\kappa_n\rangle$ and substituting into (5.36) we obtain the differential equation

$$i\hbar \dot{c}_n = e^{i(\phi - kgt^2)} \left[ c_{(n+1)} e^{i4\omega_r t(2n+1)} + c_{(n-1)} e^{i4\omega_r t(2n-1)} \right]$$

(5.43)

and solve it numerically to get $|\psi(t)\rangle_1$ and then get $|\psi(t)\rangle = U_0(t) |\psi\rangle_1$. Putting in the parameters for the order-2 reflect, and solving in the presence of gravity, we find that the reflect pulse contributes $\phi_{2r} = 0.183\pi$ to the overall phase of the interferometer every time it is applied. To see the contribution of gravity we set $g = 0$ and calculate the value of $\phi = 0.193\pi$. The difference $\Delta \phi_g = 0.01\pi$ is due to gravity during the pulse. Additionally, we would like to know how sensitive we are to an error in the intensity of the beam. We can control the intensity to better than $\pm 5\%$. The model suggests that an uncertainty in the phase of $\pm 0.05\pi$ giving $\phi_{2r} = (0.18 \pm 0.05)\pi$ per order-2 reflect. The order-1 reflect is common to both packets and so its phase contribution is negligible.

### 5.2.4 Analysis

With the necessary theoretical machinery in place, we can now focus on the experimental results and analysis. We performed 14 different interferometers with varying numbers of cycles. An interferometer curve was created for each one and then fit to obtain the phase. Fig. 5.3 shows the results of each interferometer with the associated uncertainty in the phase.

We know that each additional cycle contributes $-18\pi$. If we take the values in Fig. 5.3 and subtract $18\pi$ for each order-2 reflect, we get Fig.5.4. A linear fit combined
Figure 5.3: Graph of acquired phase for interferometers with varying numbers of cycles.

Figure 5.4: Linearized phase for interferometers with varying numbers of cycles.
with EQ. (5.32) gives

\[ \phi = A + B \times N \]  \hspace{1cm} (5.44)

\[ = A + N \left( \phi_{r2} + \frac{\hbar k_0}{m} \left[ -4k_0t + 2\gamma t^2 \right] \right) \]  \hspace{1cm} (5.45)

\[ = A + (-56.583 \pm 0.003)N \]  \hspace{1cm} (5.46)

\[ = A + (-18.011 \pm 0.001)\pi N \]  \hspace{1cm} (5.47)

Combining (5.45) and (5.47) gives

\[ \left( \frac{\hbar k_0}{m} \left[ -4k_0t + 2\gamma t^2 \right] \right) = (-18.011 \pm 0.001)\pi - \phi_{r2}. \]  \hspace{1cm} (5.48)

Putting in \( \phi_{2r} \) we find

\[ -36.357\pi + 2k_0gt^2 = (-18.19 \pm 0.05)\pi. \]  \hspace{1cm} (5.49)

Solving for \( g \) gives

\[ g = \frac{18.167\pi}{2k_0t^2} = 9.745 \pm 0.027 \text{ m}/\text{s}^2 \]  \hspace{1cm} (5.50)

where the uncertainty is almost entirely due to \( \phi_{2r} \). If we could neglect the error in \( \phi_{2r} \), the measurement of \( g \) would result in an uncertainty of \( \pm 0.003\text{m}/\text{s}^2 \) or a fractional uncertainty \( \frac{\Delta g}{g} = 3 \times 10^{-4} \).

### 5.3 Configuration 2

Configuration 2 addresses some of the issues that limit the performance of configuration 1, namely small packet separation and an error in the acquired phase that depends on how well we know the intensity of the order-2 reflect pulse. Packet separations in this configuration are twice that of configuration 1 because we use only the order-2
reflect pulse to reverse the motion of both packets. Additionally, configuration 2 is inherently more symmetric because both the upper and lower packets experience the same reflect operations. The interferometer phase is therefore less sensitive to the intensity of the reflect pulses. Despite the advantages that this configurations offers, a more complicated setup made this configuration difficult to implement and further complicated the analysis. In spite of the difficulties, we were still able to demonstrate a four-cycle vertical interferometer with high visibility.

5.3.1 Experiment

Configuration 2 is detailed in Fig. 5.5 and at first glance, it seems very similar to that of configuration 1. The first notable difference is that this interferometer sequence begins when the free-falling atoms are stationary. Due to the dynamics of the trap turn-off, the atoms are neither stationary nor free-falling when the trap is turned off. We resolve this issue by waiting 1.297 ms after the trap turn-off followed by an order-2 reflect. 1.183 ms later the trapping field is completely off and the atoms are motionless at the top of their trajectory ready for the interferometer sequence to begin.

Asymmetric split

Another difference with this configuration is the asymmetric nature of the split operation. Here we would like to split a stationary cloud into two pieces with half the atoms left at rest and the other half launched upward at $v = 2v_r$. This in principle is not possible when the atoms are stationary with respect to the standing wave because the atoms will scatter photons symmetrically. However, as demonstrated in configuration 1 with our $\pi/2$ pulse, a split operation where half the atoms are moving at the initial velocity, $v_0$ and the other half are moving at $v_0 + 2v_r$ can be
Figure 5.5: Schematic illustrating the trajectory of wavepackets in config. 2. When the atoms are stationary, an asymmetric split is performed utilizing a two-color standing wave with a difference frequency $4 \times \omega_r$ where $\omega_r = 2\pi \times 3.77$ kHz is the recoil frequency. The asymmetric split is the same as a $\pi/2$ operation in the frame where the standing wave is stationary. It can be performed by a single $34.2$ µs pulse of the traveling standing wave with intensity $\beta = 1.95\omega_r$ or a single $67$ µs pulse with $\beta = 0.99\omega_r$ and forms the superposition $\frac{1}{\sqrt{2}}(|0\rangle + |2v_r\rangle)$. The motion of the packets is made cyclical by applying order-2 reflects every $t \approx \frac{2\omega_r}{g}$. The order-2 reflects are represented by wide arrows and the split and recombination by skinny arrows. The interferometer output is represented by dashed lines.

performed for atoms that are initially moving at $-v_r$ relative to the standing wave.

Equivalently we can create a so-called traveling standing wave (TSW) by interfering two counter-propagating laser beams of slightly different frequency. The two beams interfere producing a null in the intensity that passes through the atoms at a velocity $v = 2k\Delta$ for a difference frequency $\Delta$. If we choose the difference frequency, $\Delta$ so that $\Delta = 2k_lv_r$, where $k_l$ is the wave-number of the laser, the stationary atoms in the lab frame will be moving at $-v_r$ in the frame of the standing wave. Now performing the $\pi/2$ pulse in the frame of the standing wave is identical to the $|0\rangle \rightarrow \frac{1}{\sqrt{2}}(|0\rangle + |2v_r\rangle)$ split that we desire in the lab frame. More details about how we implement the TSW
will be given later.

**Interferometer**

After the application of the split, the initially stationary atoms begin to accelerate downward, while the initially upward-launched atoms begin to slow down. After a time \( t_2 \approx \frac{2v_r}{g} \) when the bottom packet has accelerated to \(-2v_r\), the top packet is stationary at the top of its trajectory. Application of our regular order-2 three-pulse reflect operation reverses the motion of the bottom packet and leaves the motion of the top packet unchanged. Another time \( t_2 \) later, the bottom packet is now stationary while the top packet is moving downward at \(-2v_r\). We apply our order-2 reflect pulse, this time reversing the motion of the top packet. In this way the initially upward-moving packet continues to be above the initially stationary packet. The cycle is repeated a desired number of times by applying an order-2 reflect every \( t \approx \frac{2v_r}{g} \). After \( N \) cycles, the packets are recombined using our \( \pi/2 \) split pulse, but with the direction of the traveling standing wave reversed. Note that the recombination pulse replaces what would have been an even-numbered reflect.

An absorption image is taken following an 11 ms time-of-flight after recombination. The number of atoms in each output port of the interferometer is determined by fitting the clouds in the absorption image using a two-dimensional Gaussian function. The accumulated phase is related by

\[
\frac{N_0}{N} = \cos^2(\phi/2 + \alpha)
\]  

where \( N_0 \) is the number of atoms in the bottom cloud after the time-of-flight, \( N \) is the total number of atoms in both clouds, \( \phi \) is the differential phase acquired by the packets due to gravity, and \( \alpha \) is the applied phase.
5.3.2 Traveling standing wave

The traveling standing wave is used in this configuration for the split and recombination operations, which are critical components to the interferometer’s operation. An optical standing wave whose spatial interference pattern moves at speed \( v \) is created with two counter-propagating laser beams of slightly different color or equivalently, different frequency. The speed of the standing wave is given by

\[
v = \frac{\Delta}{2k_l} = \frac{\Delta \lambda}{4\pi}
\]  

(5.52)

where \( \Delta \) is the angular frequency of the relative detuning and \( k_l = \frac{2\pi}{\lambda} \) is the laser wavenumber. If \( v = \pm n v_r \) is chosen, a split operation, asymmetric in the lab frame can be performed. In our the experiment we choose \( v = v_r \) by setting \( \Delta = 4 \times \omega_r \approx 2\pi \times 15 \text{ kHz} \). The concept of using a traveling standing wave for an asymmetric split is not new. It has previously been demonstrated with condensates [48] and specifically in the context of condensate interferometry [34]. As you can imagine, the optical setup for the TSW is significantly more complex than the single beam and single mirror required for the standing waves of chapters 3 and 4, and section 5.2. The main concern is the stability of the interference pattern produced at the location of the atoms. For the case of the standing wave, stability is achieved by locating the retro-mirror on a stable mount that does not move much during the course of the interferometer sequence and by ensuring the drift in laser frequency is slow compared to the interferometer time.

The TSW is also required to be phase stable. Two criteria satisfy this requirement: the phase of the laser beams must be locked and the path that one beam follows must not change length by even tens of nanometers relative to path of the other beam. Both criteria can be met by using the same laser to generate both beams of different detuning and constraining the beams to follow the exact same path, passing through...
the same optical elements. We developed a novel way to accomplish this by using two AOMs. The phase of the standing wave is controlled with an RF phase shifter. Sweeping the phase at a constant rate gives the frequency difference required for the split operation. The two beams are kept separate by using opposite polarizations while still allowing them to traverse essentially the same path.

Double AOM

Acousto-optic modulators (AOMs) are routinely used in optical setups to turn laser beams on and off or to apply a frequency shift to a laser beam. An AOM consists of a piezoelectric transducer (PZT) in contact with a piece of glass or some other optical material. The PZT is modulated by an RF source (in our case at 80 MHz). An acoustic pressure wave is created in the glass causing a spatially varying index of refraction along the direction of sound wave propagation. Because the index of refraction varies periodically, the glass will act as a diffraction grating for a laser beam entering at an angle nearly perpendicular to the acoustic wave. Unlike a diffraction grating, because the pressure wave is moving through the glass, the diffracted beam will be Doppler shifted by an amount equal to the RF drive frequency. The ratio of light in the deflected and undeflected beams is controlled by the amount of RF power used to drive the PZT.

Our optical setup is illustrated in Fig. 5.6. We use commercially available AOMs from Crystal Technology, Inc. AOM0 is only used to adjust the overall intensity of the pulse and is not really part of the “double” AOM setup. The beam passes through AOM1 where the RF power has been adjusted to allow half the laser power to pass undeflected and the other half is deflected into the first diffraction order. The deflected beam experiences an 80 MHz frequency shift while the undeflected beam is not shifted. Both beams are directed into AOM2. The RF power to AOM2 is also set so that the laser power is divided evenly between the deflected and undeflected
Figure 5.6: Schematic showing the “double” AOM setup used to create a phase adjustable standing wave. AOM0 turns the beam on and off. AOM1 evenly divides the input laser beam between the straight-thru and deflected output beams. The deflected beam from AOM1 is shifted +80 MHz while the frequency of the straight-thru beam is not shifted. A $\lambda/2$-plate is placed in the path of the straight-thru beam to make the polarization of each beam orthogonal. Both beams enter AOM2 creating two sets of parallel beams at the output. One set is blocked. The beams of the remaining set are labeled as 1 and 2. The optical phase of beam 1 relative to beam 2 is adjusted by using the RF phase shifter. Both beams are focused and combined on a polarizing beam splitter (PBS). Another lens is used to columnate the beam and couple it to the fiber.

beams. Additionally, AOM2 is driven with the same RF source as AOM1 ensuring the a phase coherent frequency shift from both AMOs. Each input beam is diffracted giving rise to a total of four output beams or two sets of parallel beams. One set is blocked while the other will go to the experiment.

A Lorch voltage controlled RF phase shifter is placed between the RF source and AOM2. Adjusting the phase of the RF translates to a direct optical phase between the two beams. The RF phase shifter accepts a 0 to 15V control voltage and allows for a total maximum phase shift of 520°. The phase shift is approximately linear to the control voltage in the range between 5V and 10V. One unfortunate peculiarity the phase shifter is that the RF output varies by more than 1db with applied phase. The phase shifter calibration results are detailed in appendix ??.

A $\lambda/2$ plate is placed before AOM2 and rotated so that the two parallel beams that go to the experiment have orthogonal linear polarizations. The two beams are
Figure 5.7: Schematic showing the polarization decoding setup for producing a phase-coherent adjustable standing wave. The output of the fiber contains the both the phase-shifted and non-phase-shifted beams separated by polarization. The beam passes through the atoms. Both components are then rotated 45° with an optical Faraday rotator (OFR). Component 2 is blocked with a polarizing beam splitter. Component 1 is retro-reflected with a mirror and rotated again another 45° by the OFR. The incoming component 2 is now the same polarization as the outgoing component 1 and so will create an interference pattern.

focused with a lens and combined with a polarizing beam splitter. Another lens collimates the beam and couples the light into a single-mode fiber for transport to the experiment.

**Polarization decoding**

We use a single-mode optical fiber to deliver both beams to the experiment by a single-mode fiber. The two beams follow the same path and have opposite polarizations. To make the standing wave, we need to first separate the two components and and then rotate the polarization of one component to match that of the other component. The general method is illustrated in figure 5.7. The beam emerges from the fiber output coupler. The fiber is adjusted so that the phase shifted component of the beam is polarized horizontally. The beam passes through the the atoms and then into an optical Faraday rotator (OFR) with a 5 mm aperture from Optics for Research which rotates each component by a nominal 45°. A polarizer is used to block one component, while the other component passes the beam splitter, is retro reflected by the mirror
and reenters the OFR. During the second pass through the OFR, the polarization is rotated an additional $45^\circ$ allowing beams 1 and 2 to interfere at the location of the atoms. Notice the component at the location of the atoms with opposite polarization. Because the polarization is not right, its only contribution is an overall offset in the lattice potential. This does not affect the standing-wave operations. Furthermore, the interferometer is not affected by this idler beam because it contributes an equal Stark phase to both packets.

**pro et contra**

There are several attractive features of double AOM and polarization decoding setups and it is worth summarizing them here. With this setup we have gained the ability to adjust the position (or phase) of the standing wave by using an RF voltage-controlled phase shifter. The TSW for our asymmetric split can be created by ramping the phase at the correct rate. Both the shifted and unshifted components of the beam follow essentially the same path, so an overall drift in optical path-length before the atoms is common to both and does not induce an interferometer phase. Furthermore, the one-beam methodology simplifies alignment with the atoms and requires only a single stable mirror to give phase stability.

There were some undesired features of this setup too. The system of double AOMs requires a lot of effort to set up and align. The most difficult aspect of the alignment is using the PBS and lens to mode match the two beams and couple them into the fiber. Any small drift in the alignment of the beam before the AOMs resulted in having to realign the whole system. Luckily, once set, it would stay aligned for more than a week at a time. The phase shifter had a maximum shift of $520^\circ$, but as shown in appendix C, the shift was only linear in a narrow range of about $280^\circ$. Additionally, the shifter acted as unwanted voltage-variable RF attenuator which caused the laser power to waiver by about 10% as the phase was adjusted. These issues should be resolved with
a new system that uses an RF IQ modulator to drive the AOMs. Another deleterious aspect is the added magnetic field from the strong magnet in the OFR. The field from the OFR alone was about 18 G with a gradient of 3.6 G/cm at the atoms a distance of 15 cm. We constructed magnetic shielding from and 2 in iron pipe which helped to reduce the field to 2.3 G and the gradient to 0.1 G/cm. Ideally, the OFR would rotate the polarizations by exactly $45^\circ$ so that only the correct components of the light would interfere at the atoms. The measured rotation was actually $43^\circ$. Despite the negative aspects of the system, the asymmetric split still worked with a fidelity better than 0.8 and high-visibility interferometers were performed. Unfortunately, the atom-interferometric phase shift from these affects could not be entirely accounted for.

### 5.3.3 Results

Following the methods of section 5.2.2, the phase acquired for configuration 2 in one cycle is found to be

$$
\phi = \frac{\hbar}{m} (-k_0^2 t + k_0^2 \gamma t^2) \tag{5.53}
$$

where $\gamma = \frac{mg}{\hbar}$ and $k_0 = 2k_L = \frac{4\pi}{\lambda} = \frac{2mv}{\hbar}$. For $t = t_2 = \frac{2v_r}{g}$ where $2v_r = 11.7$ mm/s and $g = 9.81$ m/s$^2$ the accumulated phase is 0. The phase sensitivity for one cycle of this configuration is

$$
\Delta \phi = k_0 t_2^2 \Delta g. \tag{5.54}
$$

Assuming a shot-noise limited measurement gives

$$
\Delta g = \frac{\Delta \phi}{2k_L t_2^2} = \frac{1}{2k_L t_2^2 \sqrt{N}} = 4 \times 10^{-4} \text{m/s}^2 \tag{5.55}
$$

where $N$ is the number of atoms. For $n$ cycles $\Delta g_n = \frac{\Delta g}{n}$.

As depicted in figure 5.3.3, one, two, and four-cycle interferometers where per-
formed. We applied a known interferometer phase by stepping through the start voltage of the phase ramp during the split and recombination. Although the curves show high visibility, the measured phases do not agree with the model. The source of this disagreement is still unknown, however undesired phases could be introduced by the dynamics of the trap turn off or from the OFR required to rotate the polarization.

5.4 Method Comparison

We would now like to compare the sensitivity of each interferometer configuration to that of other methods including Bloch oscillations and atomic fountain devices. We begin with configuration 1. As found previously, the phase is given by

\[ \phi_1 = nk_L(2gt_1^2 - 4v rt_1) \] (5.56)

where \( n \) is the number of bounces, \( k_L = 2\pi/\lambda \), and \( t_1 = v_r/g \).

\[ \frac{\partial \phi_1}{\partial g} = 2nk_Lt_1^2 \rightarrow \Delta g_1 = \frac{\Delta \phi_1}{2nk_Lt_1^2} \] (5.57)

Setting the total time \( T = 2nt_1 \) and considering the signal to be shot-noise limited so that \( \Delta \phi_1 = \frac{1}{\sqrt{N}} \) where \( N \) is the total number of atoms gives

\[ \Delta g_1 = \frac{1}{k_L t_1 T \sqrt{N}} \] (5.58)

Substituting in for \( t_1 \) gives

\[ \Delta g_1 = \frac{g}{v_r k_L T \sqrt{N}} \] (5.59)

Similarly we can do the same procedure for configuration 2. I begin here with the
Figure 5.8: Interferometer curves for configuration 2 vertical interferometer. Shown are (a) one-cycle, (b) two-cycle, and (c) four-cycle interferometers.
results of Eq. (5.55), where
\[
\Delta g_2 = \frac{\Delta \phi_2}{2n k_L t_2^2} \quad (5.60)
\]
and \( t_2 = 2v_r/g \). Setting the total time \( T = 2nt_2 \) and considering a shot-noise limit, we find
\[
\Delta g_2 = \frac{1}{k_L t_2} \frac{1}{T \sqrt{N}}. \quad (5.61)
\]

Under the influence of a periodic potential and a small uniform force, the momentum of an atom will change periodically across the first Brillouin zone. This phenomenon is called Bloch oscillations. Here I briefly describe the work of Ferrari et al. [45] in order to compare with our methods.

They begin with laser cooled \(^{88}\)Sr atoms. The atoms are loaded into a vertically oriented optical lattice. Bloch oscillations occur due to the force of gravity on the atoms. They hold the atoms in the lattice for some time \( T \) and then turn off the trap to measure the population of each momentum state. By stepping through the times and comparing the output populations, they can determine the rate of oscillation. The Bloch oscillation frequency is given by
\[
\Omega = \frac{mg \lambda}{2\hbar} = \frac{\pi g}{v_r} \quad (5.62)
\]
and the sensitivity can be expressed as
\[
\Delta g_b = \frac{v_r}{\pi} \Delta \Omega \quad (5.63)
\]
If signal is shot noise limited,
\[
\Delta \Omega = \frac{1}{T \sqrt{N}} \quad (5.64)
\]
and
\[
\Delta g_b = \frac{v_r}{\pi} \frac{1}{T \sqrt{N}}. \quad (5.65)
\]
The last configuration is the atomic fountain and the phase is given by

\[ \phi_f = k_L g T^2. \]  \hspace{1cm} (5.66)

The sensitivity is then given by

\[ \Delta g_f = \frac{1}{k_L T} \frac{1}{T \sqrt{N}}. \] \hspace{1cm} (5.67)

To compare the relative sensitivities, we take the ratios \( \frac{\Delta g_1}{\Delta g_2} \), \( \frac{\Delta g_b}{\Delta g_2} \), and \( \frac{\Delta g_f}{\Delta g_2} \). We find

\[ \frac{\Delta g_1}{\Delta g_2} = \frac{t_2}{t_1} = 2 \] \hspace{1cm} (5.68)

showing that configuration 2 is twice as sensitive as configuration 1. Comparing configuration 2 to the Bloch oscillations we get

\[ \frac{\Delta g_b}{\Delta g_2} = \frac{v_r \pi}{\pi} \times k_L t_2 = 18 \] \hspace{1cm} (5.69)

and find that our method is 18 times more sensitive. Lastly we compare configuration 2 to the atomic fountain geometry and get

\[ \frac{\Delta g_f}{\Delta g_2} = \frac{1}{k_L T} \times k_L t_2 = \frac{t_2}{T} \] \hspace{1cm} (5.70)

showing that the fountain is more sensitive for \( T \) greater than \( t_2 \).

### 5.5 Summary

We have demonstrated two different configurations of compact vertical interferometers utilizing ultra-cold free-falling atoms manipulated by a pulsed optical standing wave. We show that our methods are competitive to that of gravitometers using
Bloch oscillations [45]. The optical setup for the first configuration is fairly straightforward. With this configuration, we achieve 40 interferometer cycles by applying 81 manipulation pulses. We measure an acceleration of $9.7453 \text{ m/s}^2$ with a measurement uncertainty of $\pm0.003 \text{ m/s}^2$ and a systematic uncertainty of $\pm0.027 \text{ m/s}^2$ due to the error in the order-2 reflect pulse intensity.

We developed a novel method to create a stable, phase-coherent traveling standing wave using two AOMs and some polarization tricks. We used this method to implement the asymmetric split operation required above for interferometer configuration 2. In spite of a more complicated optical setup, a multiple-cycle interferometer with high visibility was realized. This configuration provides larger phase sensitivity than configuration 1 and is also less sensitive to intensity errors of the reflect pulses.
Packing up:

Outlook for using ultra-cold atoms in precision gravimetry

The main goal of my dissertation was to implement a vertical interferometer that utilized ultra-cold atoms, was sensitive to gravity, and required a minimal amount of space for dropping. The project was very successful and real progress has been made towards the realization of a compact gravitational sensor. Through the work of this dissertation, we made several major contributions to the scientific and engineering communities.

We performed a thorough investigation of the use of an off-resonant standing
wave laser to manipulate the motion of atomic packets. We demonstrated splitting and reflecting with high fidelity and good agreement with theoretical expectations. We further improved fidelity by using a train of multiple pulses to drive the desired transitions and using a genetic algorithm to optimize the pulse shape. Through this method, we discovered many useful pulses using a three-pulse train, specifically order-1 ($|0\rangle$ to $\pm|2\hbar k\rangle$ and order-2 ($|0\rangle$ to $\pm|4\hbar k\rangle$) split operations as well as order-1 ($\pm|\hbar k\rangle \rightarrow \mp|\hbar k\rangle$), order-2 ($\pm|2\hbar k\rangle \rightarrow \mp|2\hbar k\rangle$), and order-4 ($\pm|4\hbar k\rangle \rightarrow \mp|4\hbar k\rangle$) reflection operations. Numerical inaccuracies for these pulses were generally less than 1 part in $10^4$. These high fidelity pulses and the methods used to calculate them will be very useful in all kinds of experiments and devices where precise control of atomic motion is required and other groups have already shown interest in implementing some of the pulses we made.

We have demonstrated that the high-fidelity reflect pulses generated by a pulse-shaped optical standing wave can be used to keep atoms suspended against gravity. With these pulses we were able to dynamically support a cloud of ultra-cold atoms by repeatedly applying a reflect operation which reversed the motion of the atoms as they fell. Atoms were still present after more than 130 order-1 reflects; more than 40 times the number of bounces achievable by the previous methods of other groups. A total “hang time” of 0.15 s for the order-1 reflect was achieved. We were able to perform more than 50 order-2 reflects for “hang times” approaching 0.12 s. The optical standing wave was aligned vertically by monitoring the horizontal displacement of the cloud after many bounces and a value of $g = 9.814 \pm 0.008$ m/s$^2$ was determined by optimizing the time between bounces. Additionally, using our bouncing technique to sample a region of only 50 $\mu$m, the Stern-Gerlach effect from ambient field gradients was observed and measured to an accuracy of 4 $\mu$T/m. This illustrated the possible use of bouncing condensates as a sensor for magnetic field
Two different configurations of compact vertical interferometers utilizing ultracold free-falling atoms manipulated by a pulsed optical standing wave have been demonstrated. We show that our methods, for the same times, exceed the sensitivity of gravitometers using Bloch oscillations by a factor of 18. The optical setup for the first configuration is fairly straightforward. With this configuration, we achieve 40 interferometer cycles by applying 81 manipulation pulses. We measure an acceleration of $9.7453 \text{ m/s}^2$ with a measurement uncertainty of $\pm0.003 \text{ m/s}^2$ and a systematic uncertainty of $\pm0.027 \text{ m/s}^2$ due to the error in the order-2 reflect pulse intensity.

We developed a novel method to create a stable, phase-coherent traveling standing wave using two AOMs and some polarization tricks. We used this method to implement the asymmetric split operation required above for interferometer configuration 2. In spite of a more complicated optical setup, a multiple-cycle interferometer with high visibility was realized. Configuration provides larger phase sensitivity than configuration 1 and is also less sensitive to intensity errors of the reflect pulses.

With these experiments we have demonstrated a level of atomic control that has never before been achieved.

### 6.1 Outlook

We have made much progress towards realizing a new regime for condensate interferometry. During our work, several interesting ideas have come up which we would like to test out.

It would be nice to get larger packet separations resulting in higher sensitivities. One possibility is to use higher-order splits and reflects. This would require a higher level of intensity control and timing resolution that could be accomplished with several upgrade to our apparatus.
Figure 6.1: Configuration 3 where each operation is composed of a reflect pulse followed immediately by a $\pi$ pulse. Packet separation continues to increase for half the total interferometer time.

Another way we have devised to get larger packet separations is to use a trajectory like that illustrated in Fig. 6.1, where each operation is a reflect immediately followed by a $\pi$ pulse. With each operation, the packet separation increases. The rate of packet separation is the same for this trajectory as that of the atomic fountains and will have the same sensitivity for equal times. With our current abilities, packet separations of 0.5 mm should be attainable.

Several things could be done to improve the quality and delivery of the optical standing wave/ TSW. Increasing the beam waist of the optical standing wave would make the intensity more uniform at the atoms and may help to alleviate atom losses when bouncing. This could enable us to go to longer suspension times. Another problem area was the creation of the TWS. Although the phase shifter worked, the non-ideal characteristics complicated the experiment and analysis. Currently work is underway to redesign the rf system for driving the AOM that would replace the
phase shifter. Another idea is to replace the single fiber with two fibers and actively stabilize the beam paths.

Currently, upgrades to our apparatus are planned that would further increase the development of these methods.
Rubidium
(a) **Fundamental constants**

<table>
<thead>
<tr>
<th>Name</th>
<th>Symbol</th>
<th>Value</th>
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<tbody>
<tr>
<td>Speed of light</td>
<td>$c$</td>
<td>$2.99792458 \times 10^8$ m/s</td>
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<tr>
<td>Standard gravity</td>
<td>$g$</td>
<td>$9.80665$ m/s²</td>
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<tr>
<td>Newton’s constant</td>
<td>$G$</td>
<td>$6.67428 \times 10^{-11}$ m³kg⁻¹s⁻²</td>
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<tr>
<td>Permeability of vacuum</td>
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<td>$4\pi \times 10^{-7}$ N/A²</td>
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<td>Permittivity of vacuum</td>
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<tr>
<td>Planck’s constant</td>
<td>$h$</td>
<td>$6.62606876 \times 10^{-34}$ Js</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$h = h/2\pi$</td>
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<tr>
<td></td>
<td></td>
<td>$1.054571596 \times 10^{-34}$ Js</td>
</tr>
<tr>
<td>Elementary charge</td>
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</tr>
<tr>
<td>Bohr magneton</td>
<td>$\mu_B$</td>
<td>$9.27400899 \times 10^{-24}$ J/T</td>
</tr>
<tr>
<td>Atomic mass unit</td>
<td>$u$</td>
<td>$1.66053873 \times 10^{-27}$ kg</td>
</tr>
<tr>
<td>Electron mass</td>
<td>$m_e$</td>
<td>$9.10938188 \times 10^{-31}$ kg</td>
</tr>
<tr>
<td>Bohr radius</td>
<td>$a_0$</td>
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</tr>
<tr>
<td>Boltzmann’s constant</td>
<td>$k_B$</td>
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(b) **Properties of $^{87}$Rb**

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<tr>
<td></td>
<td></td>
<td>$1.44316060 \times 10^{-25}$ kg</td>
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<tr>
<td>Nuclear spin</td>
<td>$I$</td>
<td>$3/2$</td>
</tr>
<tr>
<td>Scattering length</td>
<td>$a$</td>
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(c) **Transition properties**

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<th>Value</th>
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</thead>
<tbody>
<tr>
<td>Frequency</td>
<td>$\omega_o$</td>
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</tr>
<tr>
<td>Wavelength (vacuum)</td>
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</tr>
<tr>
<td>Lifetime</td>
<td>$\tau$</td>
<td>$26.24$ ns</td>
</tr>
<tr>
<td>Natural linewidth</td>
<td>$\Gamma$</td>
<td>$2\pi \cdot 6.065$ MHz</td>
</tr>
<tr>
<td>Recoil velocity</td>
<td>$v_r$</td>
<td>$5.8845$ mm/s</td>
</tr>
<tr>
<td>Recoil temperature</td>
<td>$T_r$</td>
<td>$361.96$ nK</td>
</tr>
<tr>
<td>Doppler temperature</td>
<td>$T_D$</td>
<td>$146$ $\mu$K</td>
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<tr>
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</tr>
<tr>
<td>Electron orbital $g$-factor</td>
<td>$g_L$</td>
<td>$0.99999369$</td>
</tr>
<tr>
<td>Fine structure $g$-factor</td>
<td>$g_J(5^2S_{1/2})$</td>
<td>$2.00233113$</td>
</tr>
<tr>
<td>Landé $g$-factor</td>
<td>$g_J(5^2P_{1/2})$</td>
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<td>Nuclear $g$-factor</td>
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<td>$1.3362$</td>
</tr>
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<td></td>
<td>$-0.0009951414$</td>
</tr>
</tbody>
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Table A.1: Constants relevant to the experiment. (a) Fundamental constants. (b) Properties of $^{87}$Rb. (c) $5^2S_{1/2} \rightarrow 5^2P_{3/2}$ transition properties
Figure A.1: $^{87}\text{Rb}$ principal transitions with hyperfine structure. Light at 780.246 nm is used for the MOT transitions whereas 780.232 nm light is used for “repumping” atoms from the $F = 1$ ground state.
We built a grating-stabilized extern-cavity diode laser from which form the standing wave used to manipulated the motion of the condensate. We used a Sanyo diode DL7140-201S rated for 80 mW of output at $\lambda = 780$. We later switched to AR coated diodes from Eagle. To isolate the laser from table vibrations we used damping grommets from EAR. Below are the technical drawings detailing the construction of the laser box and other related items.
Figure B.1: Cavity bracket
Collimation Tube Mount

2 Clearance Slots for #6

Dill Clearance and Countersink for #6 Socket Cap

Tap for 6-32

0.70

0.44

R0.29

0.75

1.00

Figure B.2: Tube mount
Figure B.3: Laser base
Figure B.4: Laser post
Figure B.5: Laser box bottom
Figure B.6: Laser box back
Figure B.7: Laser box left
Figure B.8: Laser box right
Figure B.9: Laser box top
Phase shifter from Lorch Microwave:

VP-360-80-8-S

The rf phase shifter from Lorch Microwave was used in the double AOM setup to create the traveling standing wave that was required for the asymmetric split operation.
Figure C.1: Schematic showing
Figure C.2: Schematic showing
Laser Power Changes with Phase Shifter Voltage

Figure C.3: Schematic showing


