Improvements and Applications of a Guided-Wave Bose Einstein Condensate Interferometer

John Hughson Burke
Paris, Kentucky

B.S., Centre College, 2004

A Dissertation presented to the Graduate Faculty of the University of Virginia in Candidacy for the Degree of Doctor of Philosophy

Department of Physics
University of Virginia
December, 2010
Abstract

This work was done with thesis advisor Charles Sackett. An atom interferometer using a coherent matter wave of 87 Rubidium in a Bose-Einstein condensate confined in a magnetic atom wave guide was analyzed for limitations and adapted to several measurement applications. The atom wave guide that confines the BEC wave packets and supports them against gravity provides flexibility for the apparatus and the possibility of miniaturization, but is shown to be a major limitation in its first implementation. Confinement limits stable interferometer measurement times and wave packet separations to 72 ms and 0.47 mm respectively or 96 ms and 0.29 mm depending configuration. Unstable measurements are demonstrated at 0.9 s. Confinement effects are discussed in detail including limitations and sources of error for future measurements.

A second implementation is constructed and shown to reduce the confinement limitations making 160 ms times and 0.94 mm separations possible in the 72 ms-0.47 mm configuration were it not for the now dominating limitation of atom interferometer signal noise sourced from mechanical vibrations.

The interferometer has been adapted to make several measurements. In addition to experiments to measure gravity and the dynamic polarizability of Rubidium discussed recently in previous theses from this research group, two new experiments are developed to measure rotation and the static polarizability of Rubidium in the ground state. The rotation experiment used an interferometer working in two dimensions to take advantage of the Sagnac effect. The area enclosed by the interferometer is proportional to sensitivity,
and here 0.15 mm² areas are demonstrated, making a device sensitive to rotations of the order of 100 μrad/s. Again, signal noise is the major limitation.

The static polarizability experiment involved placing a one dimensional atom interferometer in a well calibrated static electric field. A precision measurement could not be made due to insufficient optical access. Instead knowledge of the polarizability was used to partially verify the electric field calibration demonstrating the viability of the technique for future measurements. Both experiments show promise of becoming precision measurements and future improvements are discussed.
I have many people to thank and acknowledge. First I want to thank God. I prefer to think of His role in our lives as the One who starts us moving in a good direction but leaves it to us to steer ourselves and each other along the good path. I’ve worked with many people to conduct the experiments presented in this dissertation. In chronological order: Jessica Reeves, Ofir Garcia, Ken Baranowski, Benjamin Diessler, Jeremy Hughes all worked with me on the experiment and wrote their dissertations or theses using this apparatus. Jeremy and Ben in particular overlapped substantially in time with me and small sections in this dissertation involving the gravity and AC polarizability measurements were the subject of their dissertations. Robert Horne and Robert Leonard have worked with me as well and I now leave them the experiment to tend. Several undergraduate students have also worked in the laboratory at various times. Most recently Jiraphat “Joe” Tiamsuphat has been very helpful. I want to acknowledge fruitful conversations with Vanessa Leung, a PostDoc working on the differential gyroscope mentioned in the Chapter 6. I also want to acknowledge our collaboration with the University of Virginia Engineering School: specifically Professor Nathan Swami and graduate students, Adrea Robinson, Robert Hunter, and Theodore Reck. They aided in the construction of components for the polarizability experiment in Chapter 5.

My thesis advisor, Cass Sackett, deserves more than acknowledgement: he taught me more than I could list here, even if I wanted to, and he was a good friend when I needed it. On that note, thanks to all my friends in Charlottesville, especially the New
Zealand/Rugby crew, the D&D crew, the lunch crew, Scan and company, the So’F and RS folks, and I’m sure some other people I’m forgetting. I’d especially like to thank Meredith, Brent and Karen: Meredith for many reasons, and Brent and Karen, who helped me one, decrease my social ineptitude, and two, for being good friends when I needed it. I want to thank everyone I worked with through the Graduate Student Council and in the administration. I may be no politician, but it felt good to be useful. Gavin, I hope you graduate and do something worth your talent. I want to thank the Knights, especially Tony, for helping me stay nearer the light. I want to thank the teachers in my 22 years of education, especially Marshall Wilt and Phil Lockett at Centre College for teaching me my lab skills and how to channel my curiosity. I’d like to thank all of my graduate instructors, because they were all helpful in one way or another. I must thank all of Physics staff, especially the machinists and administrators. I didn’t truly realize how great they were until I moved away. Last, but certainly not least, I want to thank my family for supporting me through the years in many, many ways.
Contents

1 Introduction
  1.1 Atom Interferometry ............................................................. 2
  1.2 Bose-Einstein Condensation .................................................... 5
    1.2.1 BEC Theory ............................................................... 6
    1.2.2 BEC in the lab .......................................................... 10
  1.3 Guided Atoms ................................................................. 15
  1.4 Bragg diffraction .............................................................. 19
    1.4.1 Split ................................................................. 21
    1.4.2 Reflect ............................................................... 23
    1.4.3 Recombination ......................................................... 25
    1.4.4 Numerical Solutions ................................................... 27
  1.5 Atom Trampoline ............................................................. 31
  1.6 Interferometry ............................................................... 32

2 Confinement Effects
  2.1 Phase Gradients ............................................................ 35
  2.2 Gradient Model ............................................................. 40
  2.3 2-Reflect Interferometer .................................................. 46
  2.4 4-Reflect Interferometer .................................................. 55
  2.5 0-Reflect Interferometer .................................................. 56
2.6 Anharmonic Effects ............................................. 61
2.7 Phase diffusion ................................................. 64

3 External Wave-Guide ........................................... 66
  3.1 Construction .................................................. 66
  3.2 The Field ...................................................... 72
  3.3 Loading the Atoms .............................................. 78
  3.4 Electrical & Thermal Properties .............................. 81
  3.5 External Guide Performance ................................. 85

4 Rotation Sensing ................................................ 94
  4.1 Introduction .................................................... 94
  4.2 Extension to two dimensions ................................. 97
  4.3 Sagnac interferometer ....................................... 101
  4.4 Modeling a 3d interferometer ............................... 108
  4.5 3d model results ............................................. 116
  4.6 Sagnac Conclusion ........................................... 118

5 DC Polarizability .................................................. 122
  5.1 Theory ......................................................... 124
  5.2 AC Polarizability ............................................. 126
  5.3 DC Apparatus .................................................. 129
  5.4 Capacitor Construction Procedure .......................... 132
  5.5 Plate Spacing Measurement .................................. 135
  5.6 Analytic Field Calculation ................................... 140
  5.7 Atom Loading .................................................. 146
  5.8 Bragg Beam in Capacitor ..................................... 150
  5.9 Results ......................................................... 152
  5.10 Outlook ....................................................... 157
6 Conclusion

6.1 Summary ................................................................. 159
6.2 Future Work ............................................................ 160
  6.2.1 Polarizability ....................................................... 160
  6.2.2 Rotation .............................................................. 162
  6.2.3 Gravity .............................................................. 164

A Rubidium ................................................................. 168

B Cavity Transfer Lock .................................................. 171
  B.1 Paper Published in Rev. Sci. Inst. ............................... 171
  B.2 Program Code ........................................................ 178
  B.3 Program Explained .................................................. 187

C 3d simulation code ...................................................... 190

D ANSYS code ............................................................ 208

E Current Noise .......................................................... 210

Bibliography ............................................................... 213
List of Figures

1.1 Atom Interferometer Schematic ........................................... 3
1.2 BEC transition ............................................................... 14
1.3 Waveguide Drawing .......................................................... 16
1.4 Waveguide Trap Frequencies ............................................... 18
1.5 Cartoon of diffraction ....................................................... 20
1.6 Fidelity and Velocity Sensitivity of Bragg Operations ................. 28
1.7 Reflect Pulse Time Effects .................................................. 29
1.8 Bragg Operation Errors ...................................................... 30
1.9 Atom Trampoline ............................................................ 32
1.10 Interference Curve .......................................................... 33

2.1 Single Reflect Trajectory ................................................... 36
2.2 Single Reflect Interferometer ............................................... 37
2.3 Phase gradient profile on BEC after interferometer .................... 39
2.4 Single Reflect Interferometer Visibility and Gradient ................. 45
2.5 Asymmetric Two Reflect Trajectory ....................................... 47
2.6 Asymmetric Interferometer Visibility and Gradient .................... 50
2.7 Symmetric Two Reflect Visibility ......................................... 51
2.8 Visibility in 5 V and 2 V traps ........................................... 52
2.9 Interferometer Residual Motion Phase ................................... 54
2.10 Four Reflect Interferometer Visibility ........................................... 56
2.11 Four Reflect Residual Motion Phase ............................................. 57
2.12 Phase Noise in 4-reflect interferometer ........................................ 58
2.13 Free Oscillation Trajectory ......................................................... 58
2.14 Interferometer variance for free oscillation .................................... 59
2.15 Maximum BEC Wave-packet Separation ........................................ 61

3.1 New Guide Plates ................................................................. 68
3.2 New Guide Spacer ............................................................... 69
3.3 New Guide Assembled ............................................................ 70
3.4 New Vacuum Chamber ........................................................... 71
3.5 Picture of New Waveguide ......................................................... 72
3.6 Bragg beam geometry ............................................................ 73
3.7 Guide Current Cross-section ...................................................... 74
3.8 Z component of Linear Quadrupole Field ...................................... 76
3.9 $y$ Field Gradient Measurement .................................................. 77
3.10 Loading the External Guide ....................................................... 79
3.11 New Guide Driver Circuit ......................................................... 83
3.12 Guide Temperature Effects ....................................................... 84
3.13 External Guide Visibility .......................................................... 86
3.14 External Guide Variance .......................................................... 87
3.15 Vibration Noise Model ............................................................ 92
3.16 Differential Interferometer trajectory .......................................... 93

4.1 Square Wave Perturbation to Harmonic Oscillator ............................ 99
4.2 Sagnac interferometer trajectory ................................................. 102
4.3 Area-enclosing interference curves .............................................. 103
4.4 Area-enclosing interferometer phase measurement ........................... 104
<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.5</td>
<td>Area-enclosing Interferometer Asymmetry</td>
<td>105</td>
</tr>
<tr>
<td>4.6</td>
<td>Area-enclosing interferometer visibility</td>
<td>106</td>
</tr>
<tr>
<td>4.7</td>
<td>Trajectory Errors</td>
<td>115</td>
</tr>
<tr>
<td>4.8</td>
<td>Distorted Sagnac Trajectory</td>
<td>116</td>
</tr>
<tr>
<td>4.9</td>
<td>Simulated Moving State Asymmetry</td>
<td>117</td>
</tr>
<tr>
<td>4.10</td>
<td>Simulated Sagnac Visibility</td>
<td>119</td>
</tr>
<tr>
<td>5.1</td>
<td>AC Polarizability Schematic</td>
<td>127</td>
</tr>
<tr>
<td>5.2</td>
<td>AC Polarizability Data</td>
<td>128</td>
</tr>
<tr>
<td>5.3</td>
<td>Polarizability Experiment Schematic</td>
<td>130</td>
</tr>
<tr>
<td>5.4</td>
<td>Capacitor Schematic</td>
<td>131</td>
</tr>
<tr>
<td>5.5</td>
<td>Field Plate Drawings</td>
<td>133</td>
</tr>
<tr>
<td>5.6</td>
<td>Capacitor Vacuum Mount</td>
<td>134</td>
</tr>
<tr>
<td>5.7</td>
<td>Capacitor Picture</td>
<td>135</td>
</tr>
<tr>
<td>5.8</td>
<td>Etalon Fringe</td>
<td>137</td>
</tr>
<tr>
<td>5.9</td>
<td>Plate Spacing Measurement</td>
<td>139</td>
</tr>
<tr>
<td>5.10</td>
<td>Electric Potential Integral Contour</td>
<td>142</td>
</tr>
<tr>
<td>5.11</td>
<td>Electric Field in Capacitor</td>
<td>145</td>
</tr>
<tr>
<td>5.12</td>
<td>Positioning the BEC near the Capacitor</td>
<td>147</td>
</tr>
<tr>
<td>5.13</td>
<td>Atom Loading</td>
<td>148</td>
</tr>
<tr>
<td>5.14</td>
<td>Bragg Beam Optics for Field Plates</td>
<td>151</td>
</tr>
<tr>
<td>5.15</td>
<td>FORT Trap Diffraction</td>
<td>152</td>
</tr>
<tr>
<td>5.16</td>
<td>Interferometer Inside Capacitor</td>
<td>153</td>
</tr>
<tr>
<td>5.17</td>
<td>Polarizability Interference</td>
<td>155</td>
</tr>
<tr>
<td>5.18</td>
<td>Predicted Polarizability Interference</td>
<td>156</td>
</tr>
<tr>
<td>5.19</td>
<td>Predicted Visability for Electric Field</td>
<td>157</td>
</tr>
<tr>
<td>6.1</td>
<td>Differential Polarizability Trajectory</td>
<td>161</td>
</tr>
</tbody>
</table>
6.2 Three Conductor Geometry .................................................. 162
6.3 Differential Gyroscope ......................................................... 163
6.4 Ladder Interferometer ......................................................... 165
A.1 $^87$Rb principal transitions with hyperfine structure .................. 170
B.1 Schematic of lock system ..................................................... 173
B.2 Conditioning circuit ......................................................... 174
B.3 Lock State diagram ......................................................... 175
E.1 Quadrupole Current Noise and Capacitance ............................. 211
E.2 Quadrupole Current Noise by Control Source ......................... 211
E.3 New Waveguide Current Noise Spectrum .............................. 212
List of Tables

1.1 BEC Parameters .................................................. 15
1.2 Split Operation .................................................. 29
1.3 Reflect Operation ................................................ 31

3.1 Fit Parameters for Fig. 3.8 ...................................... 75
3.2 Loading Sequence for External Guide .......................... 81

5.1 Polarizability Values from Literature ......................... 123
5.2 Sequence for Loading Capacitor ............................... 149

A.1 Fundamental Constants ......................................... 168
A.2 Properties of $^{87}$Rb ........................................... 169
Chapter 1

Introduction

Measurement is one of the pillars of the scientific method. Experimental measurements can test new ideas; furthermore, they can open frontiers of knowledge and spur new ideas. Throughout history, measuring a quantity with higher precession has opened the door to new discoveries and new scientific knowledge. For example, Arno Penzias and Robert Wilson won the 1978 Nobel prize for confirming the Cosmic Microwave Background predicted by George Gamow, Ralph Alpher and Robert Herman in their model of a Big Bang, but more precise measurements of the radiation field revealed anisotropies that provide a new window into understanding the universe and another Nobel prize: this time for John Mather and George Smoot. The WMAP [1] satellite yielded even higher precision and the Planck Surveyor may improve our understanding still farther.

In atomic physics, we seek to understand the very small rather than the very large. With cold atoms, we look to the extreme in low energy. In very low energy systems, scientists gain a level of control that allows them to probe quantum mechanical and electrodynamics theories to such a high precision that it’s easy to claim these theories are the most rigorously confirmed theories in all of physics. Measurements with ten digits of precision almost seem routine. Measurements of atoms’ electronic transitions are reaching 17 digits of precision [2]. One might well ask “why go any farther?” The reason is that in
these new regimes new questions can be asked such as “Are the physical constants really constant?”

There is of course a secondary technological motivation for measurement. Better technology for measuring one quantity can enable new and more precise measurements of other quantities to be made. For example, measuring atomic transitions allows better clocks to be made, and better clocks in Global Positioning Satellites allow better position measurements and better topographical mapping allow geological changes such as glacial melting to be measured which in turn can test theories of climate change [3]. Progress in precision measurements and measurement technology is the motivation of this work. This thesis will explore improvements and applications of one particular measurement technique: guided-wave Bose-Einstein condensate interferometry.

1.1 Atom Interferometry

Interferometry is a powerful measurement technique. Two waves of the same medium, when caused to overlap in space and time, produce a pattern as their crests and troughs interact with each other. The interaction pattern, called interference, under the proper conditions can be used to deduce knowledge about the history of the waves. Electromagnetic waves, of which light is an example, exhibit this interference phenomenon and have been used as a medium to make sensitive measuring instruments called interferometers. One of the most famous examples is the Michelson-Morey interferometer that disproved the existence of the luminiferous ether in 1887 [4, 5].

In a twist of nature, Louis deBroglie [6] showed in 1924 that matter is also a wave medium with a wavelength \( \Lambda \) inversely proportional to its momentum \( p \) with Planck’s constant \( h \) (refer to the Appendix for constant values) as the proportionality constant:

\[
\Lambda = \frac{h}{p} \quad (1.1)
\]
Figure 1.1: Atom Interferometer Schematic

A fraction of atoms from a single beam (dotted) is deflected at point (a). These two beams are deflected back towards each other at point (b). The two beams interfere at point (c), where two output beams emerge (dashed). The output beams’ atom number is detected at point (d).

This was demonstrated with electrons shortly afterwards by Davisson and Germer [7]. In this work we will deal with atom deBroglie matter waves, which have found use in many precision experiments [8, 9]. All matter has some momentum because it has a finite temperature above absolute zero. For a free ideal gas, the average momentum of a particle of mass $m$ and temperature $T$ is

$$p = \sqrt{2\pi k_b m T}$$

where $k_b$ is Boltzmann’s constant. For everyday objects the masses and temperatures involved are such that the resulting thermal deBroglie wavelength is so small that any interference pattern would be on a scale too small to be observed. However, for microscopic masses like atoms, the interference can be detected. In an atom interferometer two “input” beams of atoms are overlapped and the interference of the deBroglie matter waves produce an interference pattern that is sensitive to the environment the two beams experienced prior to combining. In this case, the pattern is simple. When the two input beams combine, two independent output beams will emerge and the ratio of atomic flux in these output beams will change for particular changes between the relative environments the input beams experienced. Schematically this is shown in Figure 1.1.

A quantity useful in describing how this system works is the phase of the wave. The
phase of a wave measures how many oscillations the wave has undergone from some reference point. Consider the schematic atom interferometer in Fig. 1.1 in the presence of a gravitational force pointing down. The beam of atoms that is deflected up into a higher gravitational potential energy will have to slow down to conserve energy. By the time the beams recombine, the upper beam will have oscillated more than the lower beam. Its phase will have shifted and the output beam ratio will change. Knowledge of a few experimental parameters allows for a measurement of the acceleration due to gravity this way. In fact atom interferometers are among the best gravitometers [8, 9, 10, 11]

The phase $\phi$ can be calculated with knowledge of the action $S$ which is the difference between the kinetic and potential energy of a constituent particle in the wave.

$$\phi = \int \frac{S(t)}{\hbar} dt$$  

The constant $\hbar$ is Planck’s constant divided by $2\pi$. Each atom in the beam is its own particular wave. A beam of many atoms is actually comprised of many waves of possibly different wavelength and actions. Solving Eq. (1.3) and recovering the information in the action that describes what you want to measure can become difficult in many situations. In order to describe a wave with many contributing particles, it is useful to introduce the notion of coherence. Coherence comes in two related forms: temporal and spatial. Temporal and spatial coherence describe the level of sameness across the wave in time or space respectively. A laser for example is a coherent source of light waves. Each light particle (photon) is exactly the same. This makes lasers much easier to use for interferometry compared to the many wavelengths or “white” light emitted by light bulbs.

In atoms, there is always a spread of kinetic energy in any source one might use. The energy spread smears out the action and phase making an incoherent wave source for an interferometer. The spread in energy is a function of the temperature and the fraction of particles in a state of energy $\epsilon$ at temperatures $T$ in an ideal gas is given by
the Maxwell-Boltzmann distribution:

\[ f(\epsilon, T) = g_\epsilon e^{-(\epsilon - \mu)/k_bT} \]  

where the \( g_\epsilon \) is the degeneracy of the state and \( \mu \) is the chemical potential of the system, which quantifies how the energy of the system changes with particle number. What we see here is that for warm temperatures, \( k_bT \gg \epsilon \), not many particles will be in any given energy state. The particles will be distributed amongst many different energy levels. Having a colder sample is advantageous, all other parameters being equal, because it narrows the energy distribution.

1.2 Bose-Einstein Condensation

A Bose-Einstein condensate (BEC) is a special state of matter observed for the first time in the alkali elements in 1995 by Eric Cornell and Carl Wieman (Rubidium)[12], Wolfgang Ketterle (Sodium) [13], and Randy Hulet (Lithium) [14], the first three of which were later awarded the Nobel prize, but the existence of BEC was predicted decades earlier in the 1920’s by Satyendra Bose [15] and Albert Einstein [16]. The statistics of mixtures of integer angular momentum particles, called bosons, is what enables this exotic state of matter. It turns out that at a critical density and temperature bosons are forced into the ground state. Below this density a macroscopic fraction of atoms will be in the ground state and therefore share the same quantum wave function. The Boson waves will all be in phase and therefore be a coherent matter-wave. With the advances made by laser interferometry it is natural to ask whether BEC will be an improvement over thermal atoms when used in an interferometer.
1.2.1 BEC Theory

BEC is a rich topic and beyond the scope of this thesis, especially since other theses have been written on the topic from our group [17, 18]. Good treatments of the physics can be found in the book of Pethick [19], review paper of Dalfovo [20], and proceedings from the Enrico Fermi summer school of 1999 [21]. Briefly, this section will highlight a few key points while roughly following the treatments mentioned along with [22]. The Bose distribution function describes the probability of finding a boson in a state of energy \( \epsilon \) above the ground state energy at some temperature \( T \). The number in this state is

\[
 f_b(\epsilon, T) = \frac{1}{\exp[(\epsilon - \mu)/k_b T] - 1} \tag{1.5}
\]

This reduces to the M-B distribution (1.4) for high temperatures. The interesting question is “How many particles are in the ground state with \( \epsilon = 0 \) in the limit of low temperature?” In the limit of \( T \to 0 \) it must be true that \( |\mu/k_b T| \ll 1 \) and \( \mu \) must be less than than the ground state energy if we are to have a large number of total particles. The number of atoms in the ground state with \( \epsilon = 0 \) is

\[
 N_0 = \frac{1}{\exp[-\mu/k_b T] - 1} \tag{1.6}
\]

In the low temperature limit the number of atoms in the ground state becomes \( N_0 = -k_b T/\mu \), where \( \mu \) is negative because it must be less than the ground state energy. So right away we something interesting with Bose statistics; since the chemical potential is less than the ground state energy, at low temperatures it becomes energetically favorable for the system to populate the ground state. For \( N_0 \gg 1 \), it must be true that \( \gamma \equiv \exp[-\mu/k_b T] \approx 1 \). This does not disallow a lot of particles in the exited states, just that there must be some particles in the ground state.

The ground state population \( N_0 \) is the total number \( N \) minus the number in exited
states, $N_e$. The number of atoms in excited states can be found by summing over all the states. If there are $D(\epsilon)$ states with energy $\epsilon$ and the energy spacing is small enough that we can approximate the sum as an integral then

$$N_e = \int_0^\infty D(\epsilon)f_B(\epsilon,T)d\epsilon. \quad (1.7)$$

Take a free gas as an example. The density of states $D(\epsilon)$ in a volume $V$ with deBroglie wavelength $\Lambda$ is

$$D(\epsilon) = \frac{V}{\sqrt{4\pi^2}} \left(\frac{2m}{\hbar^2}\right)^{(3/2)} \sqrt{\epsilon} \quad (1.8)$$

Then the number density in the excited state can be written

$$n_e = \frac{2}{\sqrt{\pi} \Lambda^3} \int_0^\infty \frac{\sqrt{\xi}d\xi}{\gamma \exp\xi - 1} \quad (1.9)$$

for $\xi = \epsilon/k_B T$. Since $\gamma \approx 1$ and nearly constant for large $N_0$ we can ignore it, which allows the integral to be calculated. The result is:

$$n_e\Lambda^3 = \zeta(3/2) = 2.612... \quad (1.10)$$

Here, $\zeta$ is the Riemann Zeta function. When the particles are dense enough that their deBroglie wavelengths are of same order as the inter-particle spacing, a quantum critical point occurs. A macroscopic number of particles will join the ground state. Their deBroglie waves will add up into a coherent matter wave and the density will increase. The energy (or momentum) spread will become very narrow and the atoms' position will become much more localized.

For a gas of atoms trapped in a harmonic potential with trap frequencies $\omega_i$ in the $i^{th}$ direction we have states of energy $\epsilon = \hbar(\omega_x n_x + \omega_y n_y + \omega_z n_z)$ for integers $n_i$. The analog of Eq. (1.9) is

$$N_e = \int_0^\infty \frac{d n_x d n_y d n_z}{\exp \left[ (\omega_x n_x + \omega_y n_y + \omega_z n_z)\hbar/k_BT \right] - 1} \quad (1.11)$$
The number density won’t be uniform in a trap so instead we calculate just the number in
the excited state, though (1.10) still describes the critical point for the peak density. We
again are approximating the energy spacing between different \( n \) as small enough compared
to \( k_B T \) that we can integrate instead of sum the states. There is now an additional length
scale in the problem besides the deBroglie wavelength: the harmonic oscillator length
\[ a_{ho} = \sqrt{\hbar/m\omega_{ho}} \]
for \( \omega_{ho} = (\omega_x \omega_y \omega_z)^{1/3} \) the geometric mean. The integral (1.11) works
out \cite{20} to be

\[ N_e = \zeta(3)(2\pi)^{3/2} \left( \frac{a_{ho}}{\Lambda} \right)^6 \]  
(1.12)

Here \( \zeta(3)(2\pi)^{3/2} = 94.91... \) In a trapped gas, we need to cool to sufficiently low temperatures
to make the fraction \( a_{ho}/\Lambda \) small enough that the number of atoms in the excited states is
on the same order as the total number of atoms, so the transition depends on total atom
number. We can also rewrite (1.12) in terms of a critical temperature as

\[ k_B T_c = \hbar \omega_{ho} N^{1/3} \zeta(3)^{-1} \]  
(1.13)

where \( \zeta(3)^{-1} = 0.94... \)

The BEC’s made in labs are made from atoms of integer spin (in our case \(^{87}\)Rb) and
these atoms interact which muddles the waters of the analysis so far. The interaction po-
tential, \( U_{int} \), can accurately be described at low temperature by a delta function potential
with a strength proportional to the scattering length, \( a \) \cite{20}.

\[ U_{int} = \frac{4\pi\hbar^2 a}{m} \delta(r) \]  
(1.14)

To derive Schrödinger’s equation for a whole collection of atoms, one must integrate (1.14)
over all space with the wave function of all of the other atoms. For atoms held in a trapping
potential $U_{\text{trap}}$, Schrödinger’s equation takes the form

$$i\hbar \frac{\partial \psi}{\partial t} = \left[ U_{\text{trap}} + \frac{4\pi \hbar^2 a}{m} |\psi|^2 - \frac{\hbar^2 \nabla^2}{2m} \right] \psi$$  \hspace{1cm} (1.15)$$

This is the Gross-Pitaevskii equation \[20\]. The interactions change the energy proportionally to the density, $n$ given by the $|\psi|^2$ term and the scattering length. It is a non-linear differential equation in $\psi$ so can’t be solved analytically. From mean-field theory, the wave function has a separable time dependent phase term: $\psi \to \psi(\mathbf{r}) \exp \left[-i\mu t/\hbar\right]$, which eliminates the time derivative on the left hand side of (1.15) in favor of the chemical potential. For a BEC, the momentum operator, $\hat{p} = i\hbar \nabla$, is very small compared to the interaction (for many atoms) and trapping potential terms, so as a first approximation we throw out the kinetic energy term: $\hat{p}^2/2m$. If we define the length scale established by the interactions as $\xi^2 = 1/8\pi an$ (known as the healing length) then we can rewrite the condition, $4\pi \hbar^2 an/m \gg p^2/m$, for the approximation in terms of $\xi$ and the deBroglie wavelength $\Lambda = \hbar/p$ as

$$\left( \frac{\Lambda}{2\pi} \right)^2 \gg \xi^2$$  \hspace{1cm} (1.16)$$

This is the Thomas-Fermi approximation \[23, 24\], which will be used for this thesis where needed. For an interacting BEC in a harmonic trap, the approximation yields a wave function

$$n(\mathbf{r}) = |\psi|^2 = \begin{cases} \frac{m}{4\pi \hbar^2 a} [\mu - U(\mathbf{r})] & \mu > U(\mathbf{r}) \\ 0 & \text{else} \end{cases}$$  \hspace{1cm} (1.17)$$

for chemical potential

$$\mu = \frac{\hbar \omega_{ho}}{2} \left( \frac{15Na}{a_{ho}} \right)^{2/5}$$  \hspace{1cm} (1.18)$$

The chemical potential can be positive because positive (repulsive) interactions make the ground state energy larger than zero. If we define the Thomas-Fermi lengths as
\[ L_i^2 \equiv 2\mu/m\omega_i^2 \] and the interaction parameter \( g = 4\pi\hbar^2a/m \), then we have

\[
n = \frac{\mu}{g} \left[ 1 - \frac{x^2}{L_x^2} - \frac{y^2}{L_y^2} - \frac{z^2}{L_z^2} \right]
\]

(1.19)

for \( |x| < L_x \), \( |y| < L_y \), and \( |z| < L_z \). There is no density outside this region. The healing length is a function of the density so the condition for the phase transition to BEC will be spatially dependent as well. The repulsive interaction term keeps the density from going as high as one might expect without interactions. So what actually happens here all told? Atoms obeying Maxwell-Boltzmann statistics while held in a harmonic trap are cooled down through a process I will soon describe. The density profile is Gaussian. (If one puts the energies associated with a harmonic oscillator which are proportional to square position into Eq. (1.4) then a Gaussian distribution appears with widths \( w \propto k_BT \).) When the phase space density reaches the order of unity, the deBroglie wavelength of the atoms becomes large compared to the inter-particle spacing and the BEC will begin to form. The density will increase sharply as many atoms populate the low energy ground state and the density profile will be closer to the approximation in Eq. (1.19) than a Gaussian. There will always be some atoms at the edge of the BEC where the density is low that do not cross the phase transition to BEC, which will round off the sharp edge in Eq. (1.19). The harmonic oscillator and healing lengths can be used to compare BEC in different traps from different experiments.

1.2.2 BEC in the lab

Actually making BEC is difficult but can be summed up as a two step process. Step one is to laser cool the atoms in a magneto optical trap (MOT). Much can be said about laser cooling (it did win Steven Chu, Claude Cohen-Tannoudji and William Phillips Noble prizes), but much of what could be said has been discussed in depth elsewhere [25]. In summary: lasers are directed from all orthogonal directions at a dilute gas of atoms held
in vacuum. The lasers are tuned slightly red of the atomic resonance such that atoms travelling towards the beams see the light Doppler shifted onto the resonance. Atoms preferentially absorb light moving towards them and spontaneously emit a photon in a random direction. On average atoms slow down due to the momentum transfer of the absorbed photons. These now cooler atoms are collected in a magnetic trap that makes a potential energy minimum at the intersection of the laser beams. After a few seconds, a few billion atoms can be collected at a temperature of a few hundred µK. This is very cold compared to room temperature of course, but the atoms need to be cooled significantly to reach the temperatures needed for BEC.

The next step is evaporative cooling. There are several methods, but all work in the same general way. Atoms with relatively higher energy than the average of the sample are preferentially ejected. The remaining atoms will collide several times and re-thermalize (forming a new Maxwell-Boltzmann distribution). The sample will be colder, but with fewer atoms in it. This is done repeatedly until the sample reaches BEC. For us, we are left with a BEC of a few tens of thousands of atoms at a few hundred nK.

Evaporative cooling produces a coherent sample, but at the great cost of 99.999% of the atoms! The spatial and temporal coherences of a BEC are great advantages, but the great loss of signal is a large disadvantage. In addition to the signal loss, it takes much longer to produce a BEC than laser cooled samples because one has to wait for the re-thermalization through the evaporation process. The repetition rate for data acquisition will decrease as a result. If a signal is to be integrated from several data points, then BEC suffers more loss compared to other techniques. In these experiments the repetition rate is about two minutes, which is slower than other BEC experiments. For example, the group of Dana Anderson [26] report three second repetition rates. The main difference is the strength of the trapping potential. In any case, the value of using BEC needs to be evaluated for any given experiment. The best candidates for BEC interferometry experiments will be those that take advantage of or require the coherence or localization
1.2. BOSE-EINSTEIN CONDENSATION

What follows in the remainder of this section are a few details of how BEC is made in this experiment. Laser-cooled $^{87}$Rb atoms are collected in a strong spherical quadrupole trap with magnetic field

$$\mathbf{B}_{sq} = B'_x(2z\hat{z} - x\hat{x} - y\hat{y})$$  \hspace{1cm} (1.20)

and gradient $B'_x \approx 190$ Gauss/cm. This produces a potential energy of $U_{SQ} = \mu |\mathbf{B}|$ for magnetic moment $\mu = \mu_b g_F m_F$. Here $\mu_b$ is the Bohr magneton, $g_F$ is the Landé factor, and $m_F$ is the magnetic spin quantum number. For this experiment, the atoms are in the $F = 2$ hyperfine state of $5^2S_{1/2}$ with $m_F = 2$. The Landé $g_F$-factor is approximately $1/2$.

The conductors producing the quadrupole are translated via a motorized stage in order to move the trapped atoms to a second vacuum chamber about half a meter away. This is done so that the rest of the experiment can be done in a region free of the background Rb vapor needed to produce the laser-cooled sample.

In this second chamber an additional magnetic field is turned on. This bias field of strength $B_0$ rotates at frequency $\Omega$ to make

$$\mathbf{B}_0 = B_0(\hat{x}\cos \Omega t + \hat{z}\sin \Omega t)$$  \hspace{1cm} (1.21)

The result will be a magnetic field zero from the spherical quadrupole moving on an ellipse with major axis $R_d = B_0/B'_x$. The frequency $\Omega$ is chosen to be fast compared the atomic motion (about 10 Hz) but slow compared to the Larmor frequency (about 10 MHz). In this way the atoms cannot move to the zero, but their magnetic moments can still rotate in response to it. For atoms of magnetic moment $\mu$, the resulting time-orbiting potential (TOP) is proportional to the time-averaged magnitude of the magnetic field, $U_{trap} = \mu \langle |\mathbf{B}| \rangle$, which works out to be harmonic near the minimum:

$$U_{trap} = \mu B_0 + \frac{m}{2} \left( \omega_x^2 x^2 + \omega_y^2 y^2 + \omega_z^2 z^2 \right)$$  \hspace{1cm} (1.22)
1.2. BOSE-EINSTEIN CONDENSATION

with trap frequencies

\[ \omega_x^2 = \frac{\mu B_x^2}{2m_{B_0}}, \quad \omega_y^2 = \frac{\mu B_y^2}{m_{B_0}}, \quad \omega_z^2 = 2\frac{\mu B_z^2}{m_{B_0}} \]  

(1.23)

This trap was first demonstrated by Eric Cornell’s group [27] on their way to making BEC for the first time and continues to show its versatility. The trap has the advantages of reducing Majorana loss (atoms that pass through a magnetic field zero can non-adiabatically change their spin, \( m_F \) and corresponding magnetic moment causing them to fall out of the trap) and decreasing susceptibility of the trap to environmental magnetic field noise because ac fields not at the trap frequency \( \Omega \) (or an integer multiple) will average out. We use \( \Omega = 2\pi \times 11.9 \text{ kHz} \), which avoids multiples of 60 Hz electrical noise.

We use a radio frequency field to evaporatively cool the cloud in this trap. RF radiation of the order of 1 MHz is directed at the trap that is resonant with only the hottest atoms causing them to transition their spin and moment such that they leave the trap. The RF frequency is gradually decreased at a rate that allows the atoms to re-thermalize until the BEC transition is crossed.

To acquire data about the BEC we use resonant imaging. In this process, light resonant with the \( 5S_{1/2}, F = 2 \leftrightarrow 5P_{3/2}, F = 3 \) transition in \(^{87}\text{Rb}\) at 780.241 nm is directed at the atoms. The atoms absorb the light and cast a shadow onto a CCD camera. The shadow has a form \( e^{-A} \) where \( A \) is the absorption coefficient and is proportional to the atomic density. This shadow image can be fit to a Gaussian, Thomas-Fermi, or combination to determine relevant parameters. Absorption images taken of the atoms as they cross the BEC transition are shown in Figure 1.2.

The absorption data shown in Fig. 1.2 were fit to a sum of a Gaussian and a Thomas-Fermi distribution. This can tell us roughly what fraction of atoms is in the BEC. The first broad picture at 0.2 MHz is about 20% BEC already, but still appears much more Gaussian in shape. The middle picture at 0.18 MHz is about 40% BEC and one can see
Figure 1.2: BEC transition
(color) Shown here is the absorption coefficient as a function of position in a plane perpendicular to the imaging beam. The atoms have been dropped out of the TOP trap to ballistically expand for 8 ms. The frequencies next to each peak correspond to the RF evaporation frequency. The lower images are the same as the top viewed in 2D.

that the distribution has one shape in the middle and another at the edge. The last picture at 0.14 MHz is more than 60% BEC. The maximum absorption coefficient we can observe is about 3 possibly due to light leaks or resolution limits. The phase space density \( n A^3 \) changes from 3 to 60 in these pictures, and the BEC in the figure shows 60,000 atoms at 80 nK. This data is for a highly optimized BEC and was taken on April 18, 2010. Detailed parameters for typical BECs are shown in Table 1.1.
Guided Atoms

To effectively use BEC as a wave-source, one should take advantage of its temporal coherence, and work with long measurement times. To do this one must do one of two things: work in free fall or cancel gravity. The collaboration in [28] chose the former deciding to drop a BEC in a 146 meter tall evacuated tower for about one second. One might also attempt to use an orbiting satellite platform to escape gravity.

For obvious technical reasons we chose the latter: canceling gravity. We will continue to hold the atoms in the same TOP trap the BEC is made in, but engineer the trap in such a way as to make it as weak as possible thereby reducing any perturbing effect it may have on an interferometer experiment. The static spherical quadrupole (1.20) is gradually replaced with an oscillating linear quadrupole

\[
B_{Lq} = B'_{Lq}(x\hat{x} - z\hat{z}) \sin \Omega t
\]

The linear quadrupole produces a line of zero with a gradient that varies in time. A static linear quadrupole combined with the rotating bias field in Eq. (1.21) would make a line zero that rotates in a circle tracing out a cylinder. The oscillating gradient in the linear quadrupole will alter the radius of that cylinder. The phases are chosen such that

<table>
<thead>
<tr>
<th>Name</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>atom number</td>
<td>$N$</td>
<td>40,000</td>
</tr>
<tr>
<td>temperature</td>
<td>$T$</td>
<td>100 nK</td>
</tr>
<tr>
<td>deBroglie wavelength</td>
<td>$\Lambda$</td>
<td>0.6 m\mu</td>
</tr>
<tr>
<td>geometric trap average</td>
<td>$\omega_{ho}$</td>
<td>$2\pi \times 113$ Hz</td>
</tr>
<tr>
<td>harmonic oscillator length</td>
<td>$a_{ho}$</td>
<td>1.7A</td>
</tr>
<tr>
<td>chemical potential</td>
<td>$\mu$</td>
<td>$\hbar \times 1.7$ kHz</td>
</tr>
<tr>
<td>Thomas Fermi length</td>
<td>$L_x, L_y, L_z$</td>
<td>7.9, 5.5, 3.9 m\mu</td>
</tr>
<tr>
<td>peak number density</td>
<td>$n_0$</td>
<td>$40\Lambda^{-3}$</td>
</tr>
<tr>
<td>healing length</td>
<td>$\xi$</td>
<td>0.3A</td>
</tr>
</tbody>
</table>
Figure 1.3: Waveguide Drawing (taken from [18])

The linear quadrupole and rotating bias fields are created by the inner and outer conductors respectively of four horizontal coaxial 5 cm long copper rods separated at the centers by 15 mm. The conductors are supported by Boron Nitride blocks one of which is depicted as transparent here to show the arrangement of the hookup connections. The conductors on the outside are insulated from each other with kapton sheet and the apparatus is mounted in vacuum on a copper arm for heat sinking and current connections.

The linear gradient is the strongest when the rotating bias pushes the zero directly above the atoms. A short time later the gradient decreases while the bias rotates slightly. The radius to the zero is slightly larger because the gradient is slightly decreased. The total effect is to translate the zero horizontally. If the parameters are chosen such that the gradient at its maximum just cancels gravity, then the atoms will sag just under a plane of magnetic zero in a potential minimum. This oscillating linear quadrupole plus rotating bias forms the trap for the interferometer. It will confine and guide the matter-waves for the experiment.

The potential is proportional to the time averaged magnitude of the field. For the
combination of the spherical quadrupole in Eq. (1.20), oscillating linear quadrupole above, and rotating bias in Eq. (1.21) we have to 2nd order in position

\[ \langle |B| \rangle = B_0 + \left\{ \frac{B_x'}{4B_0} x^2 + \frac{B_y'}{2B_0} y^2 + \frac{B_z'}{16B_0} z^2 \right\} + \left\{ \frac{3B_{Lq}'^2}{16B_0} x^2 + \frac{B_{Lq}'^2}{16B_0} z^2 - \frac{B_{Lq}'}{B_0} z \right\} \] (1.25)

For the linear quadrupole gradient to cancel gravity \( g \) in the \( z \) direction

\[ B_{Lq}' = \frac{2mg}{\mu}. \] (1.26)

As the spherical quadrupole trap is turned off the atoms would find no confinement along the \( y \) direction. This is not exactly true because there will be some affect of the end connections, as we will see in Chapter 2. There could be an additional gradient terms as well as spatial variation of the rotating bias. To account for this we rewrite the linear quadrupole

\[ B_{Lq} = \left( B_{Lq}' - \epsilon \right) x - \left( B_{Lq}' - \epsilon \right) y - B_{Lq}' z \sin \Omega t \] (1.27)

The waveguide part of Eq. (1.25) is rewritten

\[ \langle |B_{\text{guide}}| \rangle = B_0 - \frac{B_{Lq}'}{2} z + \left[ \frac{3(B_{Lq}' - \epsilon)^2}{16B_0} + \alpha B_0 \right] x^2 + \left[ \frac{\epsilon^2}{4B_0} + \beta B_0 \right] y^2 + \left[ \frac{B_{Lq}'}{16B_0} + \gamma B_0 \right] z^2 \] (1.28)

The harmonic trap frequencies are:

\[ \omega_x^2 = \frac{2\mu}{m} \left[ \frac{3}{16B_0} \left( \frac{2mg}{\mu} - \epsilon \right)^2 + \alpha B_0 \right] \] (1.29)

\[ \omega_y^2 = \frac{2\mu}{m} \left[ \frac{1}{4B_0} \epsilon^2 + \beta B_0 \right] \]

\[ \omega_z^2 = \frac{2\mu}{m} \left[ \frac{1}{16B_0} \left( \frac{2mg}{\mu} \right)^2 + \gamma B_0 \right] \]

The classical harmonic motion of the BEC in this potential can be observed for several choices of the rotating bias strength and fit to determine the free parameters. Figure 1.4
Figure 1.4: Waveguide Trap Frequencies (modified from [17])
The black lines are fits to the corresponding frequency data in the legend. The gray line is the geometric average \( (f_x f_y f_z)^{1/3} \) from the fits. There is a shallow minimum around 15 gauss.

(adapted from [17]) shows the results. The fits give \( \epsilon / I_{Lq} = 25 \text{ mG/cm-Amp} \), \( \alpha = 0.17 \text{ cm}^{-1} \), \( \beta = 0.05 \text{ cm}^{-1} \), and \( \gamma = 0.02 \text{ cm}^{-1} \). The geometric average of the trap frequencies is \( \omega_{ho} = 2\pi \times 2.9 \text{ Hz} \) in the 10 to 20 gauss region. This makes a harmonic oscillator length \( a_{ho} = 27 \mu \text{m} \). We make a BEC of about 40,000 atoms which has a chemical potential from Eq. (1.18) of \( \mu / \hbar = 2\pi \times 10 \text{ Hz} \). We will predominantly use a 20 gauss Bias field with trap frequencies of \( \omega_x = 2\pi \times 5.7 \text{ Hz} \), \( \omega_y = 2\pi \times 1.1 \text{ Hz} \), and \( \omega_z = 2\pi \times 3.3 \text{ Hz} \). This trap should have Thomas-Fermi lengths \( L_i = \sqrt{2\mu / m\omega_i^2} \) of \( L_x = 9 \mu \text{m} \), \( L_y = 44 \mu \text{m} \) and \( L_z = 15 \mu \text{m} \). We actually observe, from a fit to the density (1.19), \( L_x = 10 \mu \text{m} \), \( L_y = 54 \mu \text{m} \) and \( L_z = 18 \mu \text{m} \) (this corresponds to a chemical potential 50% larger). The discrepancy might be due the resolution of the imaging system, which is known to be of the order of ten \( \mu \text{m} \), or a result of a deviation from the Thomas-Fermi approximation.
The adiabatic expansion from the tighter TOP trap into this one reduces the temperature to just under one nK.

1.4 Bragg diffraction

An interferometer needs two waves. We will need some form of wave-splitter to establish two independent waves. Once the waves are split we will need to bring the waves back toward each other so they overlap and interfere. To do this we will use Bragg diffraction which uses an off resonant standing wave as the diffraction grating [25, 29]. Matter waves will diffract from the grating in a very much analogous way as optical waves diffract from a conventional grating. Bragg diffraction is often used to split the atomic beams in thermal atom interferometer [30, 31, 32, 33, 34]. Since cold atom and BEC interferometry is an extension in many ways of thermal atom interferometry, it made sense to incorporate this technique into these new interferometers [35, 36, 37, 38, 39]. We have demonstrated that Bragg diffraction can be used to very efficiently split and then reflect the BEC in our experiment [40, 41, 42].

There are several ways to understand how Bragg diffraction works. To start, consider the problem found in elementary textbooks of an optical plane wave of wavelength $\Lambda$ incident on a grating with slit spacing $d$. The diffracted wave exhibits constructive interference at angles $\theta$ subject to the condition $n\Lambda = d\sin\theta$ for an integer $n$. For a matter wave incident on a standing wave, $\Lambda$ is the deBroglie wavelength and $d = \lambda/2$ for optical wavelength $\lambda$ is the slit spacing [25]. This works as long as $\Lambda > \lambda/2$. The matter waves moving off at angles $\sin\theta = 2n\Lambda/\lambda$ have effectively been split from the $n = 0$ order and have picked up transverse momentum. If the matter wave is incident on another diffraction grating a suitable time later, the final diffracted field will show enhancement of certain orders $n$ compared to others. This is just the same as an array of point sources incident on a grating as shown in the cartoon of Figure 1.5. One can also understand how this
Figure 1.5: Cartoon of diffraction
The first diffraction grating can be treated as establishing an array of point sources of waves separated by the slit separation $d$. The second grating can enhance certain wave vectors while limiting others if positioned at the correct time later.

effect would occur by realizing that atoms incident in a laser standing wave can absorb light from one direction, but only if the atom emits the photon into the other direction via stimulated emission to conserve momentum. The atom will pick up transverse momentum of $\pm 2\hbar k$ for photon momentum $k = 2\pi/\lambda$. For incident momentum $p$, the atom changes angle by $\pm 2\hbar k/p = \pm 2\Lambda/\lambda$ just as was the case above.

To fully understand the diffraction effect we must delve into the quantum mechanics of this scenario. The standing wave shifts the energy levels of an atom via the ac stark shift (or light shift). The Schrödinger equation, neglecting interactions, becomes

$$i\hbar \frac{\partial \psi}{\partial t} = \left[ -\frac{\hbar^2}{2m} \frac{d^2}{dy^2} + h\beta \cos (2ky - \alpha) \right] \psi$$

(1.30)

for a standing wave phase $\alpha$ and $\beta$ defined as.

$$\beta \equiv \frac{\Gamma^2 I_0}{4\Delta I_{sat}}$$

(1.31)
with $\Gamma = 2\pi \times 6$ MHz as the natural line width, $I_0$ as the intensity of the standing wave and $I_{sat}$ as the saturation intensity. The saturation intensity is polarization dependent: 2.504 mW/cm$^2$ for linear, 1.669 mW/cm$^2$ for circular, 3.577 mW/cm$^2$ for isotropic, all in the far detuned case.

We can use the Bloch expansion of the wave function in the plane wave basis.

$$\psi(y, t) = \sum_n c_n(t) e^{i(2nk + \kappa)y}$$ (1.32)

where $\kappa = 2\pi/\Lambda$ is the wave number for the deBroglie wavelength of an atom with some initial momentum $p_0 = \hbar \kappa$. Inserting this into the Schrödinger equation gives an infinite array of coupled differential equations for the coefficients $c_n(t)$

$$i \frac{\partial c_n}{\partial t} = \hbar \frac{2m(2nk + \kappa)^2}{2} c_n + \frac{\beta}{2} (e^{i\alpha} c_{n-1} + e^{-i\alpha} c_{n+1})$$ (1.33)

For a rigorous solution we will have to use a numerical method, but for insight into the problem consider the approximation of truncating at $n = \pm 1$ and setting the initial momentum to zero ($\kappa = 0$).

### 1.4.1 Split

The Hamiltonian for the truncated system in the plane wave basis ($|n = -1\rangle, |n = 0\rangle, |n = 1\rangle$) is

$$H = \hbar \begin{bmatrix} 4\omega_r & \frac{\beta}{2} e^{i\alpha} & 0 \\ \frac{\beta}{2} e^{i\alpha} & 0 & \frac{\beta}{2} e^{i\alpha} \\ 0 & \frac{\beta}{2} e^{i\alpha} & 4\omega_r \end{bmatrix}$$ (1.34)

where $\omega_r = \hbar k^2/2m$ is the recoil frequency ($2.36 \times 10^4 \text{ s}^{-1}$ for Rb). We hope that for the correct series of operations of this Hamiltonian we should be able to populate mostly the
$|n = \pm 1\rangle$ states with momentum $p = \pm 2\hbar k$ with very little population elsewhere from the the cartoon with gratings above. We seek the operation that couples $|0\rangle \leftrightarrow (|1\rangle + |−1\rangle)$.

Rather than the plane wave $|n\rangle$ basis, let’s project the Hamiltonian onto a more natural basis for the operations. Define the new basis as

$$|0\rangle = |0\rangle$$  \hspace{1cm} (1.35)

$$|+_\phi\rangle = \frac{1}{\sqrt{2}}\left(e^{i\phi/2}|1\rangle + e^{-i\phi/2}|−1\rangle\right)$$  \hspace{1cm} (1.36)

$$|−_\phi\rangle = \frac{1}{\sqrt{2}}\left(e^{i\phi/2}|1\rangle − e^{-i\phi/2}|−1\rangle\right)$$  \hspace{1cm} (1.37)

The phase $\phi$ will be the phase difference of the interferometer in the end. For the split operation, the initial phase doesn’t matter so we will just set $\phi = 2\alpha$, then the Hamiltonian becomes

$$H = \hbar \begin{bmatrix} 0 & \beta/\sqrt{2} & 0 \\ \beta/\sqrt{2} & 4\omega_r & 0 \\ 0 & 0 & 4\omega_r \end{bmatrix}$$  \hspace{1cm} (1.38)

We see that the state $|−_\phi\rangle$ is decoupled. The $2 \times 2$ sub matrix coupling $|0\rangle \leftrightarrow |+_\phi\rangle$ is analogous to the standard two level Hamiltonian for a monochromatic optical wave $\Omega \cos \omega t$ incident on an atom in the dipole approximation with the Rabi frequency, $\Omega = \sqrt{2}\beta$ and the detuning $\Delta = −4\omega_r$. The two level system is a well known problem [25] and has been detailed in theses from this group [18, 43] with regards to this particular problem so I won’t go into much detail here.

Because of the $4\omega_r$ term (like an effective detuning), the population in $|0\rangle$ cannot be driven completely into $|+_\rangle$. This problem can be alleviated if two pulses are used in a method demonstrated by Wu et al. [39]. The first pulse transfers population to the superposition state $(|0\rangle + |+_\rangle)/\sqrt{2}$, then some time elapses for the phase to accumulate and finally a second pulse, identical to the first, can drive the population to the $|+_\rangle$ state.

The states will evolve in time as $\psi(t) = U\psi(0)$. In this case the propagator is $U = \ldots$
$e^{iHt/h}$. The two level propagator is:

$$U = e^{i\Delta t/2} \begin{bmatrix}
\cos \frac{\Omega \tau}{2} - i \frac{\Delta}{\Omega} \sin \frac{\Omega \tau}{2} & -i \frac{\Omega}{\Omega'} \sin \frac{\Omega \tau}{2} \\
-i \frac{\Omega}{\Omega'} \sin \frac{\Omega \tau}{2} & \cos \frac{\Omega \tau}{2} + i \frac{\Delta}{\Omega} \sin \frac{\Omega \tau}{2}
\end{bmatrix}$$

(1.39)

where $\Omega' = \sqrt{\Omega^2 + \Delta^2}$ is the generalized Rabi frequency. We need to choose $\Omega' t = (2j + 1)\pi$, for an integer $j$. For $j = 0$ (the simplest case): $\Omega' = \sqrt{2}\Omega$, and $\Omega' = \sqrt{2|\Delta|}$, then $U$ becomes

$$U_p = -\frac{i}{\sqrt{2}} e^{-iz} \begin{bmatrix} -1 & 1 \\ 1 & 1 \end{bmatrix}$$

(1.40)

Waiting $|\Delta| t_w = (2m + 1)\pi$, for integer $m$ (again choose $0$ for simplest case), with no light ($\Omega = 0$) gives

$$U_w = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

(1.41)

The whole pulse train is

$$U_{\text{split}} = U_p U_w U_p = e^{-iz} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

(1.42)

which does exactly what we set out to do. This analysis gives us

$$\beta_s = 2\sqrt{2}\omega_r = 6.678 \times 10^4 \text{ s}^{-1}$$

(1.43)

$$t_p = \frac{\pi}{4\sqrt{2}\omega_r} = 23.6 \mu s$$

(1.44)

$$t_w = \frac{\pi}{4\omega_r} = 33 \mu s$$

(1.45)

### 1.4.2 Reflect

At some time, we will need to turn the atoms around to interfere the $|n = 1\rangle$ state with the $|n = -1\rangle$ state.. The easy way to accomplish this is to just let the atoms turn around
due to the natural turning point of the harmonic trap. Easy isn’t always best though. We are working in a very weak trap so turning the atoms around will take a long time and many things can go wrong in that time as we will see later. The harder technique, but the one that will allow us to troubleshoot the interferometer best, is another standing wave pulse.

In this case we will couple \( |+\rangle_\phi \) to \(|-\rangle_\phi \) and we will need a three state version Eq. (1.39). Note that
\[
\left( |+\rangle_\phi \pm |-\rangle_\phi \right) / \sqrt{2} = e^{\pm i\phi/2} |\pm 1\rangle
\]
by inverting Eq. (1.37). Therefore, we can take \( U_r = e^{iHt/\hbar} \) of the Hamiltonian in (1.38) and solve the matrix equation:
\[
\frac{1}{\sqrt{2}} \begin{bmatrix} 0 & 1 \\ -1 & 1 \end{bmatrix} = e^{iHt/\hbar} \sqrt{2} \times \begin{bmatrix} 0 \\ 1 \end{bmatrix},
\]
(1.47)

Solving this matrix equation in the case of \( \phi = 2\alpha \) (assumes the standing wave phase hasn’t changed) yields the equations
\[
0 = -\frac{i}{\Omega} \frac{\Omega'}{2} \sin \frac{\Omega' t}{2} \quad \text{(1.48)}
\]
\[
1 = e^{i\Delta t/2} \left( \cos \frac{\Omega' t}{2} + i \sin \frac{\Omega' t}{2} \right) \quad \text{(1.49)}
\]
The second equation is redundant with a third equation. The solution is:
\[
\Omega' t = 2m\pi \quad \text{(1.50)}
\]
\[
\Delta t = (2j + 1)\pi \quad \text{(1.51)}
\]
for integers \( j \) and \( m \) not equal to zero and \( j < m \) because \( |\Delta| < \Omega' \) or one of \( \Delta \) or \( \Omega \) would be imaginary. For the simplest case of \( m = 1 \) and \( j = -1 \) we have \( \Omega' t = 2\pi \) and \( \Delta t = -\pi \).
With $\beta = \sqrt{2} \Omega$ and $\Delta = -4 \omega_r$ as before, we find

$$
\beta_r = 2\sqrt{6} \omega_r = 11.56 \times 10^4 \text{ s}^{-1} \quad (1.52)
$$

$$
t_r = \frac{\pi}{2\omega_r} = 67 \text{ } \mu\text{s} \quad (1.53)
$$

Note the reflect also writes an overall phase factor $e^{-i\pi/\sqrt{2}}$ onto the $|\rangle$ just as the split operator did, but overall phases do not matter. If we put these parameters back into $U_{\text{reflect}}$, we can see what happens when we include a different standing wave phase $\alpha$. The result is

$$
\begin{bmatrix}
0 \\
1 \\
-1
\end{bmatrix} \rightarrow 
\begin{bmatrix}
0 \\
0 \\
e^{-2i\alpha}
\end{bmatrix}
$$

This is a relative phase factor of $-4\alpha$ between the $|n = 1\rangle$ and $|n = -1\rangle$ states, which could matter if the standing wave phase changes.

### 1.4.3 Recombination

After the reflect operation, if we wait until the atoms in the $|n = 1\rangle$ and $|n = -1\rangle$ states are spatially overlapped we can re-apply the split pulse. Since quantum mechanics is reversible, if there were no phase shifts due to the standing wave or the environment, then we expect that the $|+\rangle$ will populate all atoms to the $|0\rangle$ state when operated on by the split pulse. Generally the final state will be:

$$
|\psi(t)\rangle = U_{\text{split}}U_{\text{env}}U_{\text{reflect}}U_{\text{env}}U_{\text{split}}|0\rangle
$$

where $U_{\text{env}}$ is the propagator associated with any environmental perturbations that write the phase $\phi$ between the states. For the recombination split operation, we should not assume the standing wave phase is the same as the original split pulse. As with the reflect
case, a relative phase develops of $2\alpha$ (compared to the reflect’s $-4\alpha$) between the states.

The fraction of $N$ atoms ending in the $|0\rangle$ state is just the magnitude squared of the overlap:

$$N_0 \frac{N}{N} = |\langle 0|\psi_f \rangle|^2 = \frac{1}{2} \left[ 1 + \cos(\phi + 2\alpha_s - 4\alpha_r) \right]$$

(1.56)

where $\alpha_s$ and $\alpha_r$ refer to the split (at recombination) and reflect operation standing wave phases respectively. In some cases we will use two reflect operations. In this case we have.

$$N_0 \frac{N}{N} = \frac{1}{2} \left[ 1 + \cos(\phi - 2\alpha_s - 4\alpha_{r1} + 4\alpha_{r2}) \right]$$

(1.57)

for the $r1$ and $r2$ indexing the respective reflect operation phases. Note the sign change of the recombination split operation phase.

The standing wave phase $\alpha$ is established by the round trip path length of light between the atoms position to a retro reflection mirror and back to the atoms again.

$$\alpha = 2Dnk$$

(1.58)

Here, $k$ is the wave number of the light in vacuum and $D$ is the distance travelled in material with index of refraction $n$. For the interferometer to be stable and accurate, we must have knowledge of how all of these parameters change in time. We can also vary one of the parameters to test the interferometer. The standing wave laser is locked several GHz off resonance using a scanning Fabry Perot cavity and a transfer lock to a saturated absorption locked laser. See Appendix B. With this transfer lock, we can unlock the standing wave frequency momentarily, shift the frequency and thus the standing wave phase, then shift it back and re-lock it.
1.4.4 Numerical Solutions

The preceding sections are very useful for understanding the operation of the interferometer, but they are an approximation since we truncated (1.33) at \( n = \pm 1 \) and ignored initial momentum. For a complete understanding and accurate values for the pulse times in the operations, we must use a numerical approach. The brute force way is to numerically solve the coupled differential equations of Eq. (1.33) for some sufficiently large set of plane waves, \( n \), using a standard mathematical suite like MatLab. It turns out the values converge to an accuracy better than we can actually produce with technology available by \( n = \pm 4 \) or so. In this case the reflect times and intensities change slightly to \( \beta = 4.4 \omega_r \) and \( t_r = 76 \, \mu s \) instead of \( \beta = 4.9 \omega_r \) and \( t_r = 67 \, \mu s \). The numerical approach can determine the intensity and velocity sensitivity.

In order to quantify the standing wave operations, we introduce the fidelity, \( F \), as the magnitude squared of the inner product of the desired output state \( |\psi_0\rangle \) and the calculated state \( |\psi\rangle \):

\[
F \equiv |\langle \psi_0 | \psi \rangle|^2
\]  

(1.59)

Figure 1.6 (from [42]) shows how the numerically calculated fidelity of the pulses varies with intensity and initial velocity as well as data collected. The values for the power in the standing wave beam are multiplied by a calibration factor of up to 20% to overlap with prediction primarily due to miscalibration and deviations of the beam from a standing wave. From the data we can see that the reflect pulse is significantly more velocity sensitive and has a lower peak fidelity than the split pulse.

Referring to the reflect conditions of (1.51), we see that other combinations of time and intensity could work with different choice of integers \( j \) and \( m \). Figure 1.7 shows what happens when the reflect pulse is modified. The next peak of the reflect pulse occurs at a decreased intensity of \( \beta = 3.0 \omega_r \) and longer time of \( t_r = 140 \, \mu s \). This reflect gives higher fidelity, but also higher velocity sensitivity. We also see that there should exist even
1.4. BRAGG DIFFRACTION

Figure 1.6: Fidelity and Velocity Sensitivity of Bragg Operations (taken from [44])
Subfigures (a) and (b) show numerical predictions and data collected for the split operation as functions of power and velocity in units of \( v_0 = 2\hbar k/m = 11.7 \) mm/s, where \( \hbar k/m \) is the recoil speed of an atom of mass \( m \), absorbing one photon of wave number \( k \). The velocity spread in (b) corresponds to a temperature of 60 nK. Subfigures (c) and (d) are for the reflect pulse. This figure is from [42].

longer reflect pulses with higher fidelities, presumably with increased velocity sensitivity. One might be able to use this for velocity selection of atomic samples. The temperatures corresponding to the velocity selection are of the order of a few tens of nK. These fidelities then are only possible for very cold samples then, so a BEC is a natural choice if one wants to study these sort of optical effects.

Errors due to changes in velocity dominate compared to intensity errors. For the split operation, when the velocity of the BEC is nonzero, the effect is to populate the \(|n = \pm 1\rangle \) states unevenly. In fact, for atoms of mass \( m \) moving at the recoil speed \( v_r = v_0/2 = \hbar k/m \) for optical wave number \( k \), one can populate only one of the moving states as seen in Figure 1.8. One can turn this around and use a two-color standing wave such that the standing wave pattern moves with phase velocity \( v_r \) then atoms at rest in the lab frame will be moving in the standing wave frame and thus will be asymmetrically split or in
Figure 1.7: Reflect Pulse Time Effects (taken from [44])
Subfigure (a) is for a 140 $\mu$s reflect pulse. The velocity sensitivity and fidelity increase compared to the shorter 76 $\mu$s pulse. In (b), the peak fidelity is calculated for increasing pulse times. This figure is from [42].

effect launched in one particular direction.

The reflect pulse will primarily populate erroneous states when operating on atoms moving at the incorrect velocity. The lower figure in (1.8) shows how a reflect pulse can leave some atoms in the original state, as well as transfer population to the $|0\rangle$ state and even the higher lying $|n = \pm 2\rangle$ state. This was done for a 67 $\mu$s reflect pulse. Velocity errors in the reflect are particularly troubling.

A genetic algorithm described in [44] has also been used. The genetic algorithm optimized the fidelity by trying different combinations of times and intensities, then selecting the best combination as the starting point for the next set of trials. The split operation parameters: pulse times $t_p$, wait time $t_w$ and corresponding intensities $\beta$ are summarized Table 1.2.

<table>
<thead>
<tr>
<th></th>
<th>$t_p$</th>
<th>$t_w$</th>
<th>$\beta_p$</th>
<th>$\beta_w$</th>
<th>$1 - F$</th>
</tr>
</thead>
<tbody>
<tr>
<td>analytic</td>
<td>23.6 $\mu$s</td>
<td>33 $\mu$s</td>
<td>$2.83 \omega_p$</td>
<td>0 $\omega_p$</td>
<td>$5 \times 10^{-3}$</td>
</tr>
<tr>
<td>genetic</td>
<td>21.68 $\mu$s</td>
<td>36.44 $\mu$s</td>
<td>$3.03 \omega_p$</td>
<td>0.26 $\omega_p$</td>
<td>$6.7 \times 10^{-5}$</td>
</tr>
</tbody>
</table>

Note that the intensity is not zero during the wait time. The reflect is numerically optimized in the same manner. A symmetric three pulse sequence is more efficient for
1.4. BRAGG DIFFRACTION

Asymmetry Due to Split

<table>
<thead>
<tr>
<th>Velocity [v_0]]</th>
<th>[N^+(\text{+})/N^+\text{(-)}]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1.0</td>
</tr>
<tr>
<td>0.00</td>
<td>0.8</td>
</tr>
<tr>
<td>0.25</td>
<td>0.6</td>
</tr>
<tr>
<td>0.50</td>
<td>0.4</td>
</tr>
<tr>
<td>-0.50</td>
<td>0.2</td>
</tr>
<tr>
<td>-1.0</td>
<td>0.0</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Velocity [v_0]]</th>
<th>[N^-(\text{+})/N^\text{(-)}]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1.0</td>
</tr>
<tr>
<td>0.00</td>
<td>0.8</td>
</tr>
<tr>
<td>0.25</td>
<td>0.6</td>
</tr>
<tr>
<td>0.50</td>
<td>0.4</td>
</tr>
<tr>
<td>-0.50</td>
<td>0.2</td>
</tr>
<tr>
<td>-1.0</td>
<td>0.0</td>
</tr>
</tbody>
</table>

Figure 1.8: Bragg Operation Errors

The upper figure shows the asymmetry in the number of atoms placed in the \(|+1\rangle (N_+)\) and \(|-1\rangle (N_-)\) due to a split operation applied to atoms with an initial velocity in units of \(v_0 = 11.7 \text{ mm/s}\). The lower figure shows the probability of the reflect pulse operating on the \(|-v_0\rangle\) state moving at the velocity indicated to produce atom population in the desired state \(|+v_0\rangle\) the rest state \(|0\rangle\), the original state \(|-v_0\rangle\), twice the original state \(|-2v_0\rangle\), and the sum of all of these states. The numerical model is also shown.
reflecting atoms than a single pulse. The times and intensities are given in Table 1.3 for comparison.

<table>
<thead>
<tr>
<th>Reflect Operation</th>
<th>$t_1$ (µs)</th>
<th>$t_2$ (µs)</th>
<th>$\beta_1$</th>
<th>$\beta_2$</th>
<th>$1 - F$</th>
</tr>
</thead>
<tbody>
<tr>
<td>analytic</td>
<td>67</td>
<td>N/A</td>
<td>4.9 $\omega_r$</td>
<td>N/A</td>
<td>0.1</td>
</tr>
<tr>
<td>numeric</td>
<td>74</td>
<td>N/A</td>
<td>4.4 $\omega_r$</td>
<td>N/A</td>
<td>0.06</td>
</tr>
<tr>
<td>genetic</td>
<td>15.0</td>
<td>25.0</td>
<td>1.73 $\omega_r$</td>
<td>2.45 $\omega_r$</td>
<td>$1.1 \times 10^{-4}$</td>
</tr>
</tbody>
</table>

1.5 Atom Trampoline

The deviation from $F = 1$ for these operations is unobservable for one pulse. Instead we must operate many times on the BEC. In one experiment [42] we tested many reflect operations while also measuring the acceleration due to gravity $g$ in a novel way. First the BEC is released from the waveguide trap. After falling a time $t_g = v_0/g$ the atoms will have reached the correct speed for the reflect operation. After one reflect operation, if we wait $2t_g$ the BEC will have moved up, turned around and again reached $v_0$. Optimizing the atom number by varying $t_g$ effectively measures gravity if one has independent knowledge of $v_0 = 2\hbar k/m$. This is similar to work of Impens et al. [45] using interferometry. The result of this experiment is shown in Figure 1.9. In this experiment about 100 reflect operations can be made before the atoms are lost. From this data we measure $g = 9.814 \pm 0.008$ m/s$^2$ after taking into account the force due to a magnetic gradient $B' = 8.6 \pm 0.1$ gauss/m, which we measure using a Stern-Gerlach style experiment on a BEC with a mixture of magnetic states $m_F$. This extraneous field limits the measurement in a few ways. One: our measurement of it and thus our ability to correct for it is imprecise and two: the magnetic force (along with possible optical forces) tends to move the atoms away from the center of the standing wave beam. This is part of the reason that the decay of the atom signal is not uniform in the data. After about 60 reflect operations the atoms
A BEC can be operated on by about 100 reflect operations before there are too few atoms left to observe.

move into a region of non-uniform optical intensity, and the reflect operation begins to fail. This is also partly due to the beam being slightly misaligned to gravity. For more details see the thesis of Jeramy Hughes [44].

1.6 Interferometry

The essential ideas of the interferometer have now been discussed. In summary, the atoms are loaded into the waveguide with its weak confinement in the $y$ direction. In that same direction Bragg standing wave operations are made. First a split, then some time later a reflect, then that same time again later, the split again. We can test the interferometer by purposefully varying the standing wave phase $\alpha$ in Eq. (1.56). The result is the data

![Bouncing atoms](image_url)
The standing wave phase $\alpha$, is varied over $\pi$ and a fringe in the signal $N_0/N$ is observed in Figure 1.10.

The solid curve in Fig. 1.10 a fit to $N_0/N = A_0 + A \cos(2\alpha + \phi_0)$. There is a small offset $\phi_0$ indicating an additional uncontrolled phase. The amplitude $A$ of the fringe is slightly smaller than one half indicating that the interference is not perfect. In fact we will see that at longer times between pulses $A$ will further decrease until interference is no longer observable. Our goal is to have the time between operations be as long as possible because this will allow the interferometer more time to accumulate phase from the sources we might like to measure. The more phase that is accumulated the more sensitive the interferometer is as a measuring device.

In the first two chapters of this dissertation, I will go into detail about mechanisms that reduce $A$ and sources for uncontrolled phases $\phi_0$, which are often related. I will detail techniques to eliminate or reduce some of these effects and how the interferometer
sensitivity can be extended. Then in the remaining chapters I will discuss applications of this interferometer involving measuring rotations and electric fields.
Chapter 2

Confinement Effects

2.1 Phase Gradients

The major proposed advantage of guided-wave atom interferometry is that the atom waveguide, in this case a magnetic wave-guide, should allow long measurement times to be used without the requirement of dropping the atoms a long distance and thus requiring a large device. The wave-guide solves several other problems such as the loss of signal that occurs when an unconfined atomic sample is allowed to ballistically expand into a vacuum to very low density. Confinement also reduces the need to worry about gravitational gradients. Unsurprisingly however, the confinement introduces a new set of problems, and in particular magnetic gradients. In this chapter the effects of the confinement will be identified and discussed. Much of this chapter was published in [46].

To begin, consider the simple atom trajectory geometry described in Chapter 1 of a BEC wave-packet split symmetrically at the minimum of a harmonic trap into two packets. The packets move apart some distance at a speed $v_0 = 2\hbar k/m$, then reflect and return to the minimum of the potential where they are recombined and their interference observed. See Figure 2.1. The interference signal is the ratio of the number of atoms that return to rest, $N_0$, to the total number of atoms, $N$, with $N_0/N = (1 + V \cos \phi)/2$ where we can
2.1. PHASE GRADIENTS

change the phase $\phi$ by changing the relative standing wave phase $\alpha$ between the split and recombine operations.

This “single reflect” interferometer works as shown in Figure 2.2. One can see the visibility, $V$, which measures the amplitude of the interference fringe decays rapidly between 8 and 16 ms of total time. The interferometer in this configuration is far from the regime of well separated packets. Remember that one major advantage of BEC atoms wave-packets in that they are highly localized in the sense that they have an essentially finite spatial extent. The interferometer fails as the clouds just separate, meaning we can’t use this apparatus to measure localized phenomenon.

The problem is the magnetic gradient experienced by the atoms as they move away from the magnetic potential minimum. The non-uniform potential imparts a spatially varying phase shift to the packets, so that different parts of the packet interfere with different phases and the overall visibility of the interference is degraded. Several experiments have been limited by these effects [38, 40, 47].

Referring to the single reflect trajectory in Figure 2.1, one can walk through the ex-
Figure 2.2: Single Reflect Interferometer.
Plotted on the same vertical scale are the raw data and fits to several interferometer curves of $N_0/N$ versus an applied phase shift. The total time for each curve is labeled along the y-axis. The applied phase values are the voltages applied to a PZT that shifts the grating (and frequency) of an open cavity diode laser.
2.1. PHASE GRADIENTS

Experiment to see how the phase gradient appears. The packet has a finite size, \( L \) in the Thomas-Fermi limit. As the packets move from the center of the potential, the leading edge of the condensate moves to higher potential and picks up a corresponding phase shift relative to the trailing edge. When the packets recombine, the trailing edge of one packet interferes with the leading edge of the other. This creates a spatially dependent differential phase between the packets. The phase gradient can wash out the interference pattern.

Interactions have an effect here too. As the atoms separate, the mean-field effect causes atoms in one packet to experience a repulsive potential proportional to the density of the other packet. As the two packets separate, the leading edges stop interacting before the trailing edges, causing a second spatially varying phase. This gradient has the opposite sign to the magnetic potential gradient.

Both of these problems were first recognized by Olshanii and Dunjko [48], who used them to explain the limited interferometer performance observed by Wang et al. [38]. Horikoshi and Nakagawa [47] demonstrated an interferometer using a similar geometry and showed that it, too, was limited to an arm separation consistent with that imposed by the phase gradient effect. In these experiments, the gradients were large enough that the packets could not be well-separated before the visibility was reduced to nearly zero. Olshanii et al. [48] and later Stickney et al. [49] suggested strategies to extend the arm separation by making the interaction and confinement effects cancel.

In this experiment, however, the two arms of the interferometer become more separated than the point at which the cancelation effect is useful. This is because the interactions are much weaker in this experiment than in similar experiments due to the very weak magnetic potential used. For the goal of well separated wave-packets, the interaction gradient cannot be a dominating effect, since the interaction gradient stops growing once the packets stop overlapping. So if the interaction gradient is sufficiently small to avoid washing out the visibility at the point where the packets separate, the effect will remain small thereafter. In contrast, the gradient from the confining potential will continue to
2.1. PHASE GRADIENTS

Figure 2.3: Phase gradient profile on BEC after interferometer. The top row shows absorption images of the BEC (a) before the interferometer pulse sequence and (b),(c) afterwards in the case of a single reflect interferometer where a noticeable loss of visibility has occurred. (b) is the phase conjugate of (c) i.e. if (b) is the zero momentum output port, then (c) is one of the moving states. The bottom row shows the corresponding profile above it collapsed to one dimension by integrating transverse to the gradient. It is this profile that is fit to measure the gradient.

grow, and therefore can become an important limitation. An understanding of the regime of well separated wave-packets is particularly important since it is in this regime that a BEC interferometer may be most useful. The phase gradient can be observed directly. In Figure 2.3 one can clearly observe that the absorption profile after the interferometer is modulated in (b) and (c) compared to the profile just before the interferometer sequence in (a). If the images are integrated perpendicular to the modulation direction and the absorption imaging beam (in this case the horizontal $x$ direction), then the lower plots in Figure 2.3 are obtained. The integrated profile can be fit a modulated Thomas-Fermi profile to determine the frequency of the modulation and thus the gradient:

$$A(y) = A_0 + B \cos(\theta + G y) \times \min \left[ 0, 1 - \left( \frac{y - y_0}{L} \right)^2 \right]$$

(2.1)

with fit parameters $A_0, B, \theta, G, y_0$. $G = |d\phi/dy|$ is the gradient to be measured. The
Thomas-Fermi length $L$ is fixed from a measurement the cloud before the interferometer as in Figure 2.3(a). Note that in principle one could measure the phase of the modulation and use that to recover the interference pattern, but in practice this is difficult for larger separation times because the modulation frequency quickly becomes small compared to the resolution of the imaging system.

### 2.2 Gradient Model

To create a model that explains these gradients, first consider the wave functions themselves. The atomic wave packets can be represented by mean-field wave functions $\psi_1$ and $\psi_2$ which satisfy the non-linear Schrödinger equations from Eq. (1.15)

$$\left[-\frac{\hbar^2}{2m} \frac{\partial^2 \psi_i}{\partial t^2} + V(y) + g(|\psi_i(y)|^2 + 2g|\psi_j(y)|^2)\right] \psi_i = i\hbar \frac{\partial \psi_i}{\partial t}$$

for indices $i = 1, 2$ and $j \neq i$. Here $V(y) = (m\omega^2 y^2)/2$ is the confining potential and $g$ characterizes the mean-field interactions. The first interaction term is from self-interactions, i.e. the energy associated with the interactions within the wave-packet. The second interaction term describes interactions with the other wave-packet. This description neglects fluctuation effects such as phase diffusion, which are expected to be small for our experimental parameters (discussed later in the chapter). We also approximate the system as one dimensional, since the condensate is extended along the weak axis of the trap and it is phase gradients along that direction that affect the interference.

Immediately after the splitting pulse, the packets have a distribution corresponding to the initial condensate density $n_0$, given in the Thomas-Fermi limit as

$$n_0(y) = \begin{cases} \frac{\mu}{g} \left(1 - \frac{y^2}{L^2}\right) & \text{if } V(y) < \mu \\ 0 & \text{if } V(y) > \mu \end{cases}$$

(2.3)
2.2. GRADIENT MODEL

for initial chemical potential $\mu$ from Eq. 1.18 and length $L = (2\mu)/(m\omega^2)$. In our experiment, $\mu \approx 2\pi \hbar \times 15$ Hz and $L \approx 55 \mu$m. After splitting, the packets are no longer in equilibrium, so as the experiment proceeds, the wave functions will evolve in a complicated way that is computationally intensive to model.

As a simplest approximation, the internal dynamics can be neglected entirely and the packets treated as rigid bodies. This approach is justified if the duration of the interferometer is short compared to the periods of the relevant internal modes of the packets. It was used in Refs. [48, 50] and we adopt it here as well. Stickney et al.[51] introduced a less restrictive approximation allowing for quadratic phase variations across the packet, corresponding to linear expansion or contraction of the size. As seen below, we find the rigid-packet approximation to be adequate for the single-sided interferometer.

The centers of the two packets are denoted by $Y_1(t)$ and $Y_2(t)$. We define $y_1$ and $y_2$ as the positions relative to the packet centers, so that $Y_i + y_i = y$ for $i = 1, 2$. In the rigid-packet approximation, the phase acquired by packet $i$ at position $y_i$ can be expressed as

$$\phi_i(y_i) = \frac{1}{\hbar} \int_0^T \left[ V(Y_i + y_i) + \frac{1}{2} gn_0(y_i) + gn_0(Y_i - Y_j + y_i) \right] dt$$

The reduction in the atomic density due to splitting has been included explicitly.

To evaluate (2.4), we require the center of mass trajectory $Y_i(t)$. If the condensate initially has no motional excitation, then $Y_1(t) = -Y_2(t)$ and we define $Y(t) \equiv Y_1(t)$. (Non-zero initial motion will be described in the next section) After splitting at $t = 0$, the trajectory is initially described by

$$Y(t) = \frac{v_0}{\omega} \sin(\omega t) \quad (t < \tau).$$

The reflection pulse at time $\tau$ provides a velocity kick of $-2v_0$, so the subsequent trajectory is

$$Y(t) = \frac{v_0}{\omega} \{ \sin(\omega t) - 2 \sin[\omega(t - \tau)] \} \quad (\tau < t < 2\tau).$$
2.2. GRADIENT MODEL

Note that the reflection pulse provides a fixed momentum kick and not a true reflection. This introduces asymmetry to the motion, since the packet speed during the second half of the trajectory is greater than that during the first. Classically the center of mass speed will decrease as the packet moves to higher potential energy to \( v = v_0 - \delta v \). The reflected wave will have velocity \( v = -v_0 - \delta v \). At the recombination time \( T = 2\tau \), the speed has increased to \( v_0 + 2\delta v = v_0(2\cos\omega\tau - \cos 2\omega\tau) \), or approximately \( v_0(1 + \omega^2\tau^2) \) for \( \omega\tau \ll 1 \). Also, the two packets are not perfectly overlapped at recombination [49], since \( Y(T) \approx -v_0\omega^2\tau^3 \neq 0 \). These effects are small for our conditions. For instance, at \( \tau = 10 \text{ ms} \), the velocity shift is \( 5 \times 10^{-3}v_0 \) and the packet displacement is approximately \( 10^{-2}L \). The displacement could be corrected by shortening the duration of the second part of the trajectory by about 60 \( \mu \text{s} \), but we observe no effect for such small adjustments in the experiment. We therefore use equal times for simplicity.

In principle, the trajectory will also be modified by interactions between the packets. The potential energy of one packet due to the other is

\[
U(Y) = \frac{g}{2} \int n_0(y - Y)n_0(y + Y) \, dy,
\]

which has a maximum

\[
U(0) = \frac{8}{15} \frac{\mu^2 L}{g} = \frac{2}{5}\mu N.
\]

where \( N \approx 10^4 \) is the number of atoms in the initial condensate. The net interaction energy per atom is thus \( U(0)/(N/2) \approx 2\pi\hbar \times 6 \text{ Hz} \), while the initial kinetic energy is \( E_0 = mv_0^2/2 \approx 2\pi\hbar \times 15 \text{ kHz} \). The change in velocity due to the interaction between packets is thus only \( 2 \times 10^{-4}v_0 \), making the effect on the trajectory negligible.

Three terms contribute to the phase: the confinement potential, the self-interaction energy, and the intra-packet interaction energy. The phase of packet 1 due to the confining potential is

\[
\phi^c_1(y) = \frac{1}{\hbar} \int_0^T V(y_1 + Y(t)) \, dt.
\]
Since $y_1$ is constant, the final position $y$ relative to the trap center is given by $y = y(T) = y_1 + Y(T)$, so

$$
\phi_1^c(y) = \frac{1}{\hbar} \int_0^T V(y - Y_f + Y(t)) \, dt
$$

(2.10)

where $Y_f \equiv Y(T) \approx -v_0\omega^2\tau^3$. A similar expression holds for the phase of packet 2, but with $y_2 = y + Y_f$. The phase difference is thus

$$
\phi^c(y) = \frac{m\omega^2}{2\hbar} \int_0^T [y - Y_f + Y(t)]^2 - [y + Y_f - Y(t)]^2 \, dt
$$

(2.11)

$$
= \frac{2m\omega^2}{\hbar} y \left[ \int_0^T Y(t) \, dt - TY_f \right].
$$

(2.12)

Evaluation yields

$$
\phi^c(y) = -4ky \left[ 1 - 2\cos \omega\tau + \cos 2\omega\tau + (2\omega\tau \sin 2\omega\tau - 4\omega\tau \sin \omega\tau) \right].
$$

(2.13)

where we used $v_0 = 2\hbar k/m$. For $\omega\tau \ll 1$, $\phi \rightarrow 4ky(\omega\tau)^2$, which gives a gradient $d\phi^c/dy$ of 8.5 rad/L at $\tau = 10$ ms. The correction due to non-zero $Y_f$ is given by the final two terms in (2.13), which reduce to $-8ky(\omega\tau)^4$ and are thus negligible.

The phase due to the packets’ self-interactions is

$$
\phi^s(y) = \frac{g}{2\hbar} \int_0^T n_0(y_1) - n_0(y_2) \, dt
$$

(2.14)

$$
= \frac{gT}{2\hbar} [n_0(y - Y_f) - n_0(y + Y_f)]
$$

(2.15)

$$
= \frac{m\omega^2}{\hbar} yTY_f
$$

(2.16)

where we used $2\mu = m\omega^2L^2$ in the last line. As above, this effect is of order $(\omega\tau)^4$ and is thus negligible.

The phase on packet 1 due to interactions with packet 2 is

$$
\phi^i_1(y) = \frac{g}{\hbar} \int_0^{2\tau} n_0(y + 2Y - Y_f) \, dt.
$$

(2.17)
The piecewise nature of the integrand makes this term complicated. To estimate it analytically, we make several approximations. First, we neglect the small correction $Y_f$. Second, we take the packet velocity as constant during the time in which the packets are overlapped. This is reasonable since the packet separation time is $t_s = L/v_0 = 4.6$ ms, so the velocity variation $(1/2)(\omega t_s)^2v_0$ is only $5 \times 10^{-4}v_0$. Third, we ignore the small change in velocity between the outgoing and returning trajectories. All these effects can be shown to contribute to the phase at order $(\omega \tau)^4$ or higher.

What remains is

$$\phi^i_1(y) = \frac{2\mu}{\hbar} \int_0^{\min(t_1, \tau)} \left[ 1 - \frac{(y + 2v_0t)^2}{L^2} \right] dt. \quad (2.18)$$

for separation time

$$t_1(y) = \frac{L}{2v_0} \left( 1 - \frac{y}{L} \right). \quad (2.19)$$

Packet 2 experiences a similar effect, but with $y + 2v_0t \to y - 2v_0t$ in the integral and with a different separation time

$$t_2(y) = \frac{L}{2v_0} \left( 1 + \frac{y}{L} \right). \quad (2.20)$$

Evaluating these integrals yields a differential phase

$$\phi^i(y) = \frac{2\mu}{\hbar} \left\{ (t_A - t_B) \left( 1 - \frac{y^2}{L^2} \right) - \frac{2(t_A^2 + t_B^2)v_0}{L^2} \left[ \frac{2}{3}v_0(t_A - t_B) + y \right] \right\} \quad (2.21)$$

for $t_A(y) = \min(t_1, \tau)$ and $t_B(y) = \min(t_2, \tau)$. This varies with $y$ in a complicated way, but evaluated at $y = 0$ it gives

$$\frac{d\phi^i}{dy} \bigg|_{y=0} = -8k(\omega t_A)^2 \quad (2.22)$$

where here $t_A = t_B = \min(\tau, L/2v_0)$ The gradient varies quadratically with $\omega \tau$ like the confinement effect $\phi^c$, but only for short times. In our case, $d\phi^i/dy|_{\tau=t_s/2} = -1.1$ rad/L,
2.2. GRADIENT MODEL

Figure 2.4: Single Reflect Interferometer Visibility and Gradient.
The top plot shows the measured visibility as a function of the total single reflect interferometer time. The solid curve is from the theory discussed and the dashed curve is the same theory neglecting interactions. The lower plot shows the gradient increasing on the same time scale as the visibility is decreasing above. $L$ is the Thomas-Fermi length of the condensate along the guide so the values of the gradient are measured in radians per unit condensate length.
which is small but not negligible. In comparison to Eq. (2.13), we see that $\phi^c$ becomes the dominant contribution for experiments in which $\tau > t_s$ such that the packets are well separated.

Since different parts of the packets recombine with different phases, the overall visibility of the interference signal is reduced. The total number of atoms brought to rest is given by

$$N_0 = \frac{1}{2} \left[ N + \int n_0(y) \cos(\theta + \phi(y)) \, dy \right], \quad (2.23)$$

where the imperfect overlap of the final wave packets is neglected. This yields an interference visibility

$$V = \frac{1}{N} \int n_0(y) \cos \phi(y) \, dy. \quad (2.24)$$

Experimentally, the visibility and its uncertainty are measured by scanning $\theta$ and fitting the resulting signal to the form $(1 + V \cos \theta)/2$. Figure 2.4 shows a comparison of the measured and predicted visibility curves. At short times, we obtain typical visibilities of about 0.9, due to imperfections in the standing wave operations, imaging, fitting, and other time-independent experimental limitations. We therefore scale the theoretical curve by this factor. Again, good agreement is obtained, definitively showing that the phase gradient effect is the limiting factor for the interferometer performance.

The visibility is reduced by one half at $\tau \approx 6$ ms, corresponding to a gradient of 4 radians across the full packet. At this time, the centers of the clouds are 120 $\mu$m apart and the packets are just separated.

### 2.3 2-Reflect Interferometer

It is evident from the above discussion that the dominant contribution to the phase gradient comes from the $\int Y \, dt$ term in $\phi^c$, Eq. (2.13). This explains why the gradient can be reduced by using a trajectory in which $Y$ alternates sign so that $\int Y \, dt$ is close to zero.
2.3. 2-REFLECT INTERFEROMETER

Figure 2.5: Asymmetric Two Reflect Trajectory.
Here $2\tau_1$ and $2\tau_2$ are the “arm” times, the time a wave-packet spends on a side of the potential.

Using two reflect pulses will cause the interferometer to be more symmetric. To demonstrate, consider the asymmetric two-reflect interferometer with leg duration $\tau_1$ in the first half and $\tau_2$ in the second half shown in Fig. 2.5. The packet trajectory is given by

$$Y(t) = \frac{10}{\omega} \times \begin{cases} 
\sin \omega t & (t < \tau_1) \\
\sin \omega t - 2 \sin \omega (t - \tau_1) & (\tau_1 < t < 2\tau_1 + \tau_2) \\
\sin \omega t - 2 \sin \omega (t - \tau_1) + 2 \sin \omega (t - 2\tau_1 - \tau_2) & (t > 2\tau_1 + \tau_2)
\end{cases}$$

(2.25)

Again, the effects of interactions on the trajectory are neglected.

The calculation of $\phi^c$ and $\phi^s$ can be performed just as for the single-sided case, yielding

$$\phi^c = 4ky \left\{1 - 2 \cos u_2 + 2 \cos(u_1 + 2u_2) - \cos(2u_1 + 2u_2) - 2(u_1 + u_2) \left[\sin 2(u_1 + u_2) - 2 \sin(u_1 + 2u_2) + 2 \sin u_1\right]\right\}$$

(2.26)
2.3. 2-REFLECT INTERFEROMETER

and

\[ \phi^s = 4ky(u_1 + u_2) \left[ \sin 2(u_1 + u_2) - 2 \sin(u_1 + 2u_2) + 2 \sin u_1 \right] \] (2.27)

for \( u_i = \omega \tau_i \).

The intra-packet effect is again complicated, and because the phase cancels to lowest order, the simplifying approximations used in the single-sided case are not permissible. We can, however, calculate analytically the phase gradient \( d\phi^i/dy \) evaluated at \( y = 0 \).

From (2.17),

\[ \frac{d\phi^i}{dy} = \frac{g}{\hbar} \int_0^T \frac{dm_0}{dy} \left| \begin{array}{l} 2Y - Y_f \end{array} \right| dt = -\frac{2\mu}{\hbar L^2} \int_C 2Y - Y_f dt \] (2.28)

where the integral is over times when the packets are overlapped, given by \( n_0(2Y - Y_f) > 0 \). The gradient in packet two is equal and opposite to this, making the total phase gradient

\[ \frac{d\phi^i}{dy} = -\frac{2m\omega^2}{\hbar} \int_C 2Y - Y_f dt. \] (2.29)

Evaluation for the trajectory of (2.42) yields

\[ \frac{d\phi^i}{dy} = k(\omega t_s)^2 \left\{ \left( u_1^2 - u_2^2 \right) \left[ 1 - u_1 u_2 - \frac{7}{12} \left( u_1^2 + u_2^2 \right) \right] - u_1 u_2 (u_1 + u_2)^2 \right\} \] (2.30)

to order \( (\omega \tau)^6 \). As before, \( t_s = L/v_0 \) is the packet separation time, and we have assumed \( \tau_1, \tau_2 > t_s/2 \). If \( \tau_1 > t_s/2 \) and \( \tau_2 < t_s/2 \), then the gradient reduces to

\[ \frac{d\phi^i}{dy} = -2k\omega^2 \left( t_s^2 - 4\tau_2^2 \right) \] (2.31)

to leading order.

We performed an experiment in which we fixed \( \tau_1 = 10 \text{ ms} \) and varied \( \tau_2 \). In general, we observed modulation in the output packets as in Fig. 2.3, from which we obtain an estimated gradient by fitting to Eq. (2.1). The results are shown in Fig. 2.6. The solid curve in (b) shows the total predicted gradient \( d\phi/dy \) at \( y = 0 \). It is seen that when
2.3. 2-REFLECT INTERFEROMETER

\( \tau_1 = \tau_2 \), the gradient largely cancels, as expected. Figure 2.6(a) shows how the resulting visibility is recovered. We note that the cancellation effect is very much analogous to the spin echo phenomenon known in magnetic resonance studies and the photon echo in the de Broglie wave interferometer of [52].

With a symmetric interferometer \( \tau_1 = \tau_2 \equiv \tau \), the measurement time can be significantly extended compared to a single-sided interferometer, as seen in Fig. 2.7. The total measurement time \( T \) is increased by a factor of six, while \( \tau \) and the packet separation are increased by a factor of three. The separation distance of 0.4 mm is now substantially greater than the packet size.

In the symmetric case, the theoretical value of the phase is, to leading order,

\[
\phi = -\frac{64}{3} k y (\omega \tau)^6. \tag{2.32}
\]

The predicted visibility is shown as the solid curve in Fig. 2.7, and is seen to extend to considerably longer times than experimentally observed. However, at such long times, the assumption that \( \omega T \ll 1 \) no longer holds, meaning that the rigid-packet model is not justified. Stickney et al. have analyzed the case of a symmetric double-sided interferometer using a model in which variation of the packet length is included to lowest order [51]. They find a leading-order phase of

\[
\phi = -88k y (\omega \tau)^6, \tag{2.33}
\]

resulting in the dashed curve in Fig. 2.7. This is in better agreement with the experiment, though it still predicts a larger coherence time than observed.

We attempted to weaken the longitudinal confinement by using smaller bias fields, which should increase the measurement times available. This will also increase the confinement in the transverse directions making a highly elongated wave packet. The transverse frequencies increase faster than the longitudinal frequency so the overall confinement increases and with it the density. In Figure 2.8 one can see that the measurement times
2.3. 2-REFLECT INTERFEROMETER

Figure 2.6: Asymmetric Interferometer Visibility and Gradient.
While holding the first arm time, $\tau_1$, at 10 ms, the second arm time, $\tau_2$, is varied. The visibility peaks (a) while the gradient (b) goes to zero.
Figure 2.7: Symmetric Two Reflect Visibility.
The measured visibility falls decays sooner than both the model discussed in the text (solid) and the model of J. Stickney (dashed), which includes some of the packet dynamics. Both theories were again scaled by 0.9 as in the one reflect case.

actually decrease for traps with lower longitudinal frequency.

The explanation for this effect might come from the fact that the theory thus far is one-dimensional. The experiment is not in the one-dimensional regime precisely, so packet dynamics will occur in the transverse directions as well. Since the different directions are coupled by the interactions, the dynamics in the $y$-direction will have components at the transverse frequencies [20], which could be significant and non-trivial on the time scales observed here. Also, if the standing wave is misaligned to the principle axis of the potential, then the atoms might move slightly in the transverse directions relative to each other and then miss each other slightly at recombination or worse pick up a phase gradient related to the faster transverse frequency. The possibility of packets missing each other at
recombination will become more evident in the case of the free oscillation interferometer discussed later. The effect of sampling transverse trap frequencies will be discussed in detail in the next chapter within the context of purposely operating in two dimensions. It is clear from this data that working in the overall weakest possible trap is beneficial. This makes some sense, the less perturbative the trap beyond cancelling gravity the better.

In the experiment, images of the atoms do not exhibit gradients like those seen in Fig. 2.3 as the visibility of the double-sided interferometer falls to zero. Instead, the recombination appears to be uniform. This might also be explained by transverse dynamics, if a transverse gradient develops along with the longitudinal one. This would mask the longitudinal gradient, since the absorption imaging technique averages over phase variations that are parallel to the probe beam.
There could also be of course some as yet unknown effect associated with the change in the guide potential and whatever this might be could perhaps explain the lower performance of the interferometer compared to what the models suggest. Still, measurement times of up to 72 ms and arm separations of 420 $\mu$m are a significant improvement.

In the above analysis we treated the case of atoms at rest at the bottom of the potential. In the case that there is some displacement, $\delta$ from the trap minimum, or some initial velocity, $\epsilon$, one could repeat the analysis by modifying Eq. (2.42) with the center of mass of all the clouds motion as

$$Y(t) \rightarrow Y(t) + \frac{\epsilon}{\omega} \sin \omega t + \delta \cos \omega t$$

(2.34)

valid for all times $t$. Continuing through the analysis gives

$$\phi_U = -\frac{8k\epsilon}{\omega}(\omega \tau)^3 + 16k\delta(\omega \tau)^4 + O(\omega \tau)^5$$

(2.35)

There are no additional gradient terms, but there is a uniform phase growing in time. There is also an offset phase from the laser pulses. Each pulse imparts a phase proportional to $\alpha = kD$, where $D$ is the distance to the mirror. If there is a overall center of mass motion $Y_{COM}$ as in Eq. (2.34), then following the introductory chapter’s analysis in Eq. (1.57), the total phase from the laser is

$$\phi_L = k[Y_{COM}(0) - 2Y_{COM}(\tau) + 2Y_{COM}(3\tau) - Y_{COM}(4\tau)]$$

(2.36)

which to lowest order is

$$\phi_L = \frac{2k\epsilon}{\omega}(\omega \tau)^3 - 4k\delta(\omega \tau)^4 + O(\omega \tau)^5$$

(2.37)

Therefore, the total phase one might expect for an center of mass motion is just the sum
The phase, $\phi_0$, from a fit of $N_0/N = y_0 + A \cos (2\pi x + \phi_0)$ for a 5 V bias, two-reflect interferometer as a function of time. The curve is a fit to the data of $\phi_0 = BT^3 + CT^4$ that yield $\epsilon = 0.29$ mm/s and $\delta = 0.65$ mm of these two contributions

$$\phi_0 = -6k\frac{\epsilon}{\omega}(\omega\tau)^3 + 12k\delta(\omega\tau)^4 + O(\omega\tau)^5 \quad (2.38)$$

We should be able to observe this effect. From a fit to the phase offset data in Fig. 2.9 one can find a residual speed of $\epsilon = -1.3 \pm 0.4$ mm/s and displacement of $\delta = -0.6 \pm 0.5$ mm. These values are larger than we observe by about a factor of four. Also, the slight dip in the data at shorter times, suggest there might be contributions to the phase at lower orders in time. This would be the case if the pulses were not timed perfectly.
2.4 4-Reflect Interferometer

A natural question to ask is whether adding more reflect pulses improves the performance. If we follow the same analysis as in the previous sections we can easily find what the phase for such a geometry should be. For an initial displacement from the trap center $\delta$ and initial velocity $\epsilon$ we find the phase in a 4-reflect interferometer to be

$$\phi_4 = -12k \frac{\epsilon}{\omega}(\omega \tau)^3 + 48k \delta(\omega \tau)^4 - 1225k \frac{\epsilon}{\omega}(\omega \tau)^5 + \frac{4492}{3}k \delta(\omega \tau)^6 - \frac{1024}{3}ky(\omega \tau)^6 + O(\omega \tau)^7$$

(2.39)

In comparison to Eq. (2.32) and Eq. (2.38), one can see all of the pre-factors are larger. The gradient term is 16 times larger in this case so you would expect $\tau$ to be reduced by $16^{1/6} = 1.59$ to keep a working interferometer. The time between reflect pulses will be reduced from the 2-reflect case, however, the total time is twice as long for a given $\tau$ so one might reach longer total times despite the gradient increase.

If one compares Fig. 2.10 to 2.7, one can see the 4-reflect interferometer does work at longer times than the 2-reflect even though $\tau$ decreased from 18 ms to about 12 ms, a factor of 1.5 that is very much in agreement with the prediction. Despite decreasing $\tau$, the total time for which the interferometer is useful increased from 72 ms to about 96 ms.

The constant phase terms also increased so we should see larger offset phases. We see this as well. Compare Figures (2.11) and (2.9) and you’ll see that offset phase from residual motion approximately doubles. The phase changes sign indicating that either the displacement of the velocity have switched sign. Note for the longest times in the 4-reflect case it’s possible that the phase is accumulating so fast in time that it becomes difficult to determine what fringe the interferometer is on.

Another interesting feature of the 4-reflect interferometer is that as it decays, the phase noise dramatically increases. One measure of noise is the reduced $\chi^2$, which measures the spread of points away from the fit of $N_0/N$. It takes the sum of the squares of the difference between the points and the fit and divides it by the degrees of freedom, which takes into
2.5 0-Reflect Interferometer

It should be clear from the previous results that the reflect pulse is problematic. The velocity errors imposed by the reflection pulses are preventing the interferometer from reaching longer times so the natural next step is to attempt an interferometer with no

account the number of fit parameters as well as the number of points.

In Fig. 2.12, the reduced $\chi^2$ is plotted against the visibility for the two and four reflect interferometers. One can see that the phase noise in the four reflect case is four to five times larger than the two-reflect case. There will be more discussion about phase noise and its sources Chapter 3.

2.5 0-Reflect Interferometer

Figure 2.10: Four Reflect Interferometer Visibility
This interferometer displays visibility at extended total times, however the time between reflect pulses $2\tau$ must be reduced.
Four Reflect Interferometer Phase

Figure 2.11: Four Reflect Residual Motion Phase
The residual motion is about twice as large here as compared to the 2-reflect case and opposite in sign.

reflections. Instead of reflect pulses we will just allow the atoms to complete a full harmonic oscillation in the guide, as in Fig. 2.13. Insofar as the guide potential is harmonic, the trajectories for the two half-cycles will be balanced and any phase gradients should cancel precisely. Even if the packets evolve in a complicated way, the evolution should be symmetric and thus any resulting phases should cancel.

However, over the oscillation period of 0.9 s, many external noise sources can impart an uncontrolled phase difference to the packets as we were beginning to see in the four-reflect case. The interferometer output becomes effectively random. We were therefore unable to observe a controlled interference signal as in the previous experiments, but rather used the run-to-run variations in the output as a measure of the interference visibility. The presence of large fluctuations indicates that interference is occurring, while an unvarying
Phase Noise in 2 and 4 reflect interferometers

Figure 2.12: Phase Noise in 4-reflect interferometer

Free Oscillation Trajectory

In this case there are no reflects so there is no symmetry breaking.
output indicates the visibility of the interference is zero.

To obtain long measurement times, the standing wave beam must be well aligned to the guide axis, to avoid having the packets miss each other in the transverse directions. In Fig. 2.14, we show the variance of $N_0/N$ observed as the beam angle is varied, plotted against the measured packet displacement. The clear peak indicates that interference is present. Assuming the fluctuations represent an underlying distribution $(1 + V \cos \theta)/2$ with random $\theta$, the variance $\Delta^2$ is related to the visibility $V$ by $\Delta^2 = V^2/8$, thus suggesting $V = 0.3$. No fluctuations were observed when the atoms were allowed to oscillate for only one half of the axial period.

The technique of using a freely oscillating trajectory was also demonstrated by Segal et al. [53] and by Horikoshi et al. [50]. In both cases, fluctuations similar to ours were
observed at the longest interferometer times. Horikoshi and Nakagawa were also able to operate their experiment at a higher $\omega$ and achieve a stable output phase, which supports the hypothesis that the fluctuations are an effect of low frequency external noise, such as vibrations of the table on which the experiment is mounted. Recently Segal et al. [54] partially recovered the phase using statistical methods.

Without control of the overall phase, it is not possible to make use of this result in a practical measurement. However, it seems likely that this is a technical problem that is solvable through various stabilization methods. This is one advantage of the reflect pulse. It allows the interferometer to be scaled from zero time so that problems can be identified. A free-oscillation interferometer really needs a similar scaling feature if these long times are to be attained.

To put these results in context, one can define a figure of merit $\chi$ for an atom interferometer as the time integral of the wave packet separation. This is proportional to the accumulated phase for measurement of a linear gradient field such as gravity, and for a given atom velocity, it is also proportional to the Sagnac phase in a loop geometry. Previous results for our experiment were limited to $\chi = 6 \, \mu\text{m}\cdot\text{s}$ [40], and other approaches have yielded $\chi \lesssim 4 \, \mu\text{m}\cdot\text{s}$ [55, 50] with controlled phase and $\chi = 66 \, \mu\text{m}\cdot\text{s}$ with uncontrolled phase [53]. The improvements reported here give controlled phase values of $\chi = 15 \, \mu\text{m}\cdot\text{s}$ in the case of the two-reflect interferometer. Because the packet separation is smaller in the four-reflect case, even though it uses longer times, $\chi$ is slightly reduced to $14 \, \mu\text{m}\cdot\text{s}$. With an uncontrolled phase the free oscillation interferometer reached $1600 \, \mu\text{m}\cdot\text{s}$. In comparison, free space interferometers reach $\chi \approx 500 \, \mu\text{m}\cdot\text{s}$ with a controlled phase [10, 11]. Recently, advances by E. Su et al. [56] using atoms that were laser cooled but not evaporated to BEC, and manipulated in a wave guide similar to the one presented here, by standing waves also similar to the ones presented here achieved phase stable measurement times up to one second, but for packet separations of only $0.41 \, \mu\text{m}$ ($\chi = 0.41 \, \mu\text{m}\cdot\text{s}$). The largest figure of merit they achieved was $\chi = 1.8 \, \mu\text{m}\cdot\text{s}$ in a configuration with separations
2.6. ANHARMONIC EFFECTS

Through all of the previous analysis, we’ve assumed that the magnetic potential is perfectly harmonic. In this section, we will approximate the effect of an anharmonic term in the
potential of the form

\[ U = m \left( \frac{\omega_0^2}{2} y^2 + \frac{\alpha}{3} y^3 + \frac{\beta}{4} y^4 \right) \quad (2.40) \]

I will follow the method of successive approximation laid out in Landau and Lifshitz [57]. Mainly, for the equation of motion (EOM), \( \ddot{y} + \omega_0^2 y = -\alpha y^2 - \beta y^3 \), we assume a solution of the form \( y = y_0 + y_1 + y_2 \), where \( y_0 = A \sin (\Omega t) \), and \( \Omega = \omega_0 + \omega_1 + \omega_2 \). Note Landau uses a cosine instead of a sine, but we will use the initial condition of initial velocity rather than displacement.

We will treat the anharmonic terms one at a time. For the cubic contribution place \( y = y_0 + y_1 \) and \( \Omega = \omega_0 + \omega_1 \) into the EOM, and solve the resulting differential equation for \( y_1 \). Ignoring terms of order two in \( \alpha, \beta, \omega_1, y_1 \), and combinations, one can find

\[ y_1 = -\frac{\alpha A^2}{2 \omega_0^2} - \frac{\alpha A^2}{6 \omega_0^2} \cos (2\Omega t) \quad \text{and} \quad \omega_1 = 0 \quad (2.41) \]

Note, that the cubic contribution effectively displaces the minimum of the trap as evident in the first constant term. This constant term affects both clouds equally and thus cancels.

The initial condition \( \partial y/\partial t(t = 0) = v_0 \) gives \( A = v_0/\omega_0 \). To find the resulting phase we simply take the trajectory in Eq. 2.42 and alter it to conform to the added \( y_1 \) term. We find

\[ Y(t) = \begin{cases} 
Y_1 & (t < \tau) \\
Y_2 & (\tau < t < 3\tau) \\
Y_3 & (t > 3\tau)
\end{cases} \quad (2.42) \]
2.6. ANHARMONIC EFFECTS

for

\[ Y_1 = \frac{v_0}{\omega_0} \sin[\omega_0 t] - \frac{\alpha v_0^2}{6 \omega_0^4} \cos[2\omega_0 t] \]

\[ Y_2 = Y_1 - \frac{2v_0}{\omega_0} \sin[\omega_0 (t - \tau)] - \frac{\alpha (2v_0)^2}{6 \omega_0^4} \cos[2\omega_0 (t - \tau)] \]

\[ Y_3 = Y_1 + Y_2 + \frac{2v_0}{\omega_0} \sin[\omega_0 (t - 3\tau)] - \frac{\alpha (2v_0)^2}{6 \omega_0^4} \cos[2\omega_0 (t - 3\tau)] \] (2.43)

We can then integrate just as in eq. 2.12 to get the phase

\[ \phi^c(y) = \frac{m}{\hbar} \int_0^{4\tau} \left\{ \frac{\omega_0^2}{2} \left[ [y - Y_f + Y(t)]^2 - [y + Y_f - Y(t)]^2 \right] + \frac{\alpha}{3} \left[ [y - Y_f + Y(t)]^3 - [y + Y_f - Y(t)]^3 \right] \right\} dt \] (2.44)

where \( Y_f = Y(4\tau) \). The BEC is made at the minimum of the potential where the anharmonic effect is very small, thus we can treat the wave-packet as a Thomas-Fermi distribution as we have done before. The self interactions then is the same as in Eq. (2.16).

From this we get a first order correction to the phase of

\[ \phi_3 = \frac{32k}{3} \left( \frac{v_0}{\omega_0} \right)^2 \frac{\alpha}{\omega_0^3} (\omega_0 \tau)^2 + \frac{272k}{9} \left( \frac{v_0}{\omega_0} \right)^2 \frac{\alpha}{\omega_0^2} (\omega_0 \tau)^4 + 16ky^2 \frac{\alpha}{\omega_0^2} (\omega_0 \tau)^4 \] (2.45)

We see additional phase terms, and worse a quadratic phase gradient term. To see if this could be a large effect, we can use the numerical model of the waveguide conductors in [58] to find that \( \alpha \approx 1 \) rad/mm-s². For an 80 ms interferometer, this produces an extra phase over 100 radians but the gradient term is smaller. Compared to the center of the wave-packet the edges are only shifted by 2 milliradians. We do not observe such large phase offsets as predicted by this model so something is either wrong with this model or the model of [58].

Now we treat a potential quartic contribution (ignoring cubic now). Following the
Phase Diffusion

2.7. Phase Diffusion

Another effect of confinement concerns how the interactions in and of themselves can spoil the interference. The atoms, while in the superposition state, can experience phase diffusion [47, 50]. The number of atoms, in each packet will fluctuate by $\sqrt{N}$ from mea-
surement to measurement for $N$ total atoms. Since the self-interaction energy depends on atom number there will be an energy difference and thus a phase difference that fluctuated from measurement to measurement. The phase noise after time $T$ is

$$\phi_\mu = \frac{\mu T}{\hbar \sqrt{N}} \quad (2.49)$$

for chemical potential $\mu$ in Eq. (1.18). $\mu = 2\pi \times 15$ Hz for the experimental conditions here. Once $\phi_\mu = \pi$, visibility will be lost. For 40,000 atoms, this makes an upper limit time of about six seconds. This is obviously far in excess of the times available presently, but I point it out for completeness.

There is a limit to how weak the confinement can be both fundamentally and practically. The fundamental limit, described in [59], is

$$\sum_i \omega_i^2 \geq \frac{mg^2}{\mu |B|} \quad (2.50)$$

for mass $m$, local gravitational acceleration $g$, magnetic moment $\mu$, and magnetic field $B$. The requirement that the harmonic trap support the atoms against gravity sets a lower limit to what the sum of the trap frequencies can be. The practical limit is that the atom density can only be so low before the inference signal is unobservable for a particular imaging system. There is then a lower limit on the phase diffusion rate and thus an upper limit on the times achievable. This will likely be the limit for the sensitivity of an atom interferometer, but clearly this technique has a long way to go before this is a problem.
Chapter 3

External Wave-Guide

3.1 Construction

The confinement effects demonstrate that weaker axial confinement is better. To that end, a new magnetic potential was engineered. To do this required a significant change to the apparatus. The one Hertz axial harmonic confinement was thought to come from electrical connections at the end of the guide so the simplest method to decreasing the axial confinement was to make the guide physically longer to move those connections further away from the region of interest. Since the first guide is held in vacuum, the glass chamber had to be changed.

We had two options. One was to make the chamber larger to accommodate the new guide structure. The second was to use an external guide and engineer a smaller chamber to fit inside it. The advantage to an external guide structure is that the guide is easily accessible if we want to move it around between different experimental setups and level it to gravity. An external guide also allows for other internal components for additional experiments. There is a limit to how small the vacuum chamber can be made and still hold the ultra-high vacuum needed. This is because any outgassed material needs to have an unrestricted path to the vacuum pumps. To accommodate the vacuum chamber, the
guide had to be larger in the transverse directions as well. This is a disadvantage because higher currents and power dissipation are required.

On balance an external guide seemed like the best option. The new guide structure is made from solid Boron Nitride from a company named Accuratus. Boron Nitride is useful here because of its thermal properties. First, it is a good thermal conductor so that heat can be removed from the magnetic-field-producing wires which will be dissipating more power than before. Second, BN has a very low thermal expansion coefficient. This is important because if the structure changes shape during the experiment, then the interferometer may lose some of the symmetry. Even if the structure changes only over longer times the potential shot-to-shot fluctuations and drifts would be undesirable.

Figures 3.1, 3.2, 3.3, and 3.4 show the new guide structure and the chamber. The Boron Nitride came with the outer dimensions pre cut. The plates have 2 channels cut into them such that three layers of six 12 gauge Litz wires will fit into them. Litz wire was chosen for its superior electrical properties at the 12 kHz frequencies used. The wires for both the linear quadrupole and the bias fields were positioned in the guide assembly together very carefully under tension with the guide assembly plates placed end to end. The wires were glued to the assembly using thermally conducting epoxy. The epoxy used in not viscous in its liquid form so it could seep between the wires and fill any spaces. Note that the channel was machined slightly deeper than needed for three layers to allow room for an epoxy layer on top. After the epoxy hardened the two plates could be folded together and the spacers positioned to form the final product shown in Fig. 3.5. The layout is such that the guide can be folded like a clamshell around a vacuum chamber for relatively easy installation. The vacuum chamber is an all quartz design constructed by Hellma. This arrangement also allows room for the moving spherical quadrupole coils to operate unchanged from the previous experimental apparatus.

The quartz is not anti-reflection coated, so the Bragg beam needs to operate at an angle to the vacuum window to prevent etalloning in the 4.5 mm thick quartz walls.
Figure 3.1: New Guide Plates
Units are in inches. The overall length of 10 inches is about 5 times longer than the first guide structure.
Figure 3.2: New Guide Spacer
The spacer has a hole in it for Bragg beam optical access. Units are in inches.
Figure 3.3: New Guide Assembled
This figure shows how the guide parts fits together. The extra holes are for mounting the apparatus to the optical table and for mounting possible optical components or heat-sinks to the guide.

Figure 3.6 shows the important beam paths for the Bragg laser. At an incident angle of 45°, the first parallel reflected beam will be displaced from the principle beam by 3.4 mm. Also, the transmitted beam from one plate will be 1.5 mm displaced from the incident ray or 3 mm displaced going through the entire chamber. This is important since etalloning in the vacuum chamber wall creates an interference pattern that is, one, undesirable for imaging and, two, creates a high contrast interference pattern in the standing wave pulses. That interference pattern was discovered to be a major limitation in the first generation interferometer because it caused the standing wave pulses to work inconsistently. Also
The vacuum chamber is built by Hellma using quartz plates polished and then bonded together. Note the fragile point is the intersection with the small 12.7 mm tube to the rectangular box that is the science region. The height is 19 mm shown in B-B. The plates are 4.5 mm thick leaving at least 9 mm of room inside the chamber for internal components. The chamber is not anti-reflection coated. All units are in mm.
3.2 The Field

We can derive the magnetic field for the currents shown in Fig. 3.7. Using the Biot-Savart law, one can easily derive the field at the origin due to one infinitely long current carrying wire located at the center of each square with directions indicated and magnitude $I_0$. For

Figure 3.5: Picture of New Waveguide
The old waveguide is shown at the top for comparison.

note that the first reflection coefficient for quartz is 9.2% for polarization perpendicular to the incident plane and 0.8% for parallel polarization, at 45° incidence.
3.2. THE FIELD

Figure 3.6: Bragg beam geometry
The Bragg beam geometry produces several reflected beams when it passes through the vacuum chamber. The parallel reflected beam is $r = 3.4$ mm from the principle beam. The transmitted beam will be $t = 1.5$ mm displaced from the incident ray for each vacuum wall it passes through.

If we include a current density $NI/(2c)^2$ and then integrate over the box we just find the total field is Eq. (3.1) multiplied by the number of wires $N$. Then we can include the time dependence of the current $I = I_0 \cos \omega t$ to find the bias field:

$$B_0 = -\frac{\mu_0 I_0}{\pi a} \hat{z}$$

If we include $NI/(2c)^2$ and then integrate over the box we just find the total field is Eq. (3.1) multiplied by the number of wires $N$. Then we can include the time dependence of the current $I = I_0 \cos \omega t$ to find the bias field:

$$B_0 = \frac{\mu_0 NI_0}{\pi a} (\hat{z} \cos \omega t + \hat{x} \sin \omega t)$$

so we see the magnetic field rotates in a circle. The magnitude of the magnetic field is

$$B_0 = \frac{\mu_0 NI_0}{\pi a}$$
3.2. THE FIELD

In cross section the currents are distributed in a square configuration. The bias fields consist of four current regions (dark shading) centered on a square of side \(2a = 24\) mm, here labeled 1 and 2. The linear quadrupole current (light shading) is inside of the bias field on a rectangle of sides \(2a \times 2b\) with \(2b = 12\) mm. Each current region consists of a set of nine wires in a square of side \(2c = 6\) mm. The direction of the current in the linear quadrupole is labeled with an \(\odot\) for current coming towards the viewer and \(\otimes\) for current moving away.

We will need 12 Amps to form a 20 gauss bias field as we had in the previous guide. The linear quadrupole isn’t symmetric in the \(x - z\) plane so is a little more complicated to work out. For a thin wire at each location and current directions in Fig. 3.7 we find a field of

\[
B_{LQ} = \frac{\mu_0 I}{\pi} \frac{4ab}{a^2 + b^2} (x \hat{x} - z \hat{z})
\]  

(3.4)

near the origin. Now we integrate Eq. 3.4 over the actual size of the current carrying regions in Fig. 3.7 i.e. integrate \(a\) from \(a - c\) to \(a + c\) and likewise for \(b\). This is in any
### 3.2. THE FIELD

#### Table 3.1:

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>$A_0$ [gauss/cm-amp]</th>
<th>$A_1$ [gauss/cm$^2$-amp]</th>
<th>$A_2$ [gauss/cm$^3$-amp]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y = -35$ mm</td>
<td>$-7.2 \pm 0.02$</td>
<td>$6.0 \pm 0.003$</td>
<td>$-3.6 \pm 0.0001$</td>
</tr>
<tr>
<td>$y = 0$ mm</td>
<td>$-7.2 \pm 0.02$</td>
<td>$6.5 \pm 0.003$</td>
<td>$-3.2 \pm 0.0001$</td>
</tr>
<tr>
<td>$y = +35$ mm</td>
<td>$-6.8 \pm 0.02$</td>
<td>$5.9 \pm 0.003$</td>
<td>$-2.9 \pm 0.0001$</td>
</tr>
</tbody>
</table>

standard integral table. We find

$$
\mathbf{B}_{LQ} = \frac{\mu_0 NI}{4\pi c^2} \ln \left[ \frac{1 + q}{1 - q} \right] (x\hat{x} - z\hat{z}),
$$

(3.5)

where

$$
q = \frac{8abc^2}{(a^2 + b^2)^2 + 4c^4}
$$

(3.6)

What is really needed from this linear quadrupole field is a gradient in the $z$ direction strong enough to support the atoms against gravity.

The magnetic field of a prototype with $a = 8$ mm, $b = 5$ mm and $c = 3$ mm using 3 amps dc was carefully measured with a gauss meter using the Hall effect. The results are in Figure 3.8 with fit parameters summarized in Table 3.1. The expected value for the gradient term, $A_0$, from Eq. 3.5 is 7.29 gauss/cm-amp and the measured values are $7.2 \pm 0.02$ gauss/cm-amp so the calculation presented in good agreement with measurement.

Note the slight change in the gradient with changes in $y$ indicate this new design is still not perfectly uniform in $y$. The final guide has $a = 15$ mm, $b = 9$ mm and $c = 3$ mm and has a field gradient of 2.07 gauss/cm-amp according to (3.5). To cancel gravity, a gradient of 31 gauss/cm is needed therefore we need 15 Amps of current. For this final product, we can use the atoms to make a much more sensitive measurement. By gradually decreasing the current in the quadrupole and monitoring the atoms position, a minimum current of 15 Amps is required to keep the atoms from falling, again just as predicted.
3.2. THE FIELD

Figure 3.8: Z component of Linear Quadrupole Field
The magnetic field of the linear quadrupole as a function of the vertical z direction. Curves were taken at several longitudinal positions, y, a few centimeters from center. This was done for a current of 3 Amps dc in a prototype with $a = 8\text{ mm}$, $b = 5\text{ mm}$ and $c = 3\text{ mm}$. The data for the fits are shown in Table 3.1.

The purpose of this new waveguide is to create a flatter longitudinal potential. The slight changes in the measured z gradient at different y positions indicate this new design still has some y curvature. Just as was done for the gradient in the z direction above, one could just move the gauss meter probe along y and attempt to measure the gradient. Fortunately, or unfortunately depending on how you look at it, this was not feasible because the field in that direction is too small compared to the background field to be measured by the gauss meter with any accuracy. A rough estimate places the gradient at about 10 milligauss/cm-amp.

An indirect measurement can be made that takes advantage of Maxwell equation $\nabla \cdot \mathbf{B} = 0$. For this technique the Hall probe was inserted into the wave guide along the
Figure 3.9: y Field Gradient Measurement
On the left is a diagram showing the relevant geometry for acquiring the data on the right. The gauss meter probe (bold line) measures magnetic field at a point $r, \theta$ parallel to the $\hat{n}$ unit vector, which makes an angle $\alpha$ to the radial vector. The data can be averaged to find the $y$ component of the field.

$y$ direction such that it would measure fields in the $x-z$ plane. Referring to Fig. 3.9, the probe was then rotated about $y$, by an angle $\theta$ on a circle of radius 3 mm. The probe makes an angle of $\alpha = 33^\circ$ to the tangent of the circle due to the mount used for the Hall probe.

The linear quadrupole should generally have the form $\mathbf{B} = ax\hat{x} + bz\hat{z} - (a + b)y\hat{y}$, with $x = x_0 + r \cos(\theta)$ and $z = z_0 + r \sin(\theta)$. Here, $x_0$ and $z_0$ mark the displacement of the center of the circle from the magnetic field zero of the linear quadrupole. We are interested in the quantity $a + b$, which for a pure linear quadrupole should be zero. The gauss meter probe will measure $\mathbf{B} \circ \hat{n}$ where $\hat{n} = \cos(\alpha)\hat{r} + \sin(\alpha)\hat{\theta}$ is the unit vector pointing normal the probe’s surface. The probe will then measure as a function of $\theta$:

$$
\mathbf{B} \circ \hat{n}(\theta) = a(\cos \alpha \cos \theta - \sin \alpha \sin \theta)(x_0 + r \cos \theta) \\
+ b(\cos \alpha \sin \theta + \sin \alpha \cos \theta)(z_0 + r \cos \theta)
$$

(3.7)
If we average (3.8) over $\theta$, we find

$$\langle \mathbf{B} \cdot \mathbf{n} \rangle_{\theta} = \frac{1}{2} r \cos \alpha (a + b) \quad (3.8)$$

The data in Fig. 3.9 has an average of 6.85 milligauss/amp which corresponds to a gradient in $y$ of 50 mg/cm-amp when plugged into (3.8) along with $r$ and $\alpha$. Note, one can also fit the data to a form $a_1 + a_2 \cos \theta + a_3 \sin \theta + a_4 \cos 2\theta + a_5 \sin 2\theta$. The coefficient $a_1$ is the same as the average (3.8). At 15 amps, the gradient could be as high as 0.7 gauss/cm or 47 milligauss/cm-amp. Compare this to the old guide which had a $y$ gradient of 25 milligauss/cm-amp, but the old guide ran at 50 amps so there was a 1.3 gauss/cm gradient. There is almost a factor of two improvement in the $y$ gradient, and one should think that the deviation of the bias field ($\alpha$, $\beta$, and $\gamma$ in Eq. (1.30)), would similarly decrease.

To find the resulting trap frequency in the longitudinal direction note that $\omega_y/\omega_x \approx \sqrt{4/3} B'_y/B'_x$ from Eq. (1.30). $B'_x \approx B'_z$ from above. We find $\omega_y \approx 2\pi \times 0.2$ Hz. This is about 5 times flatter than the measured value of the previous guide, different than the factor of 2 indicated above because the old guide trap frequency was modified by bias fields as well as a $y$-gradient. Again, the atoms are the best detector available. In Fig. 3.10(d) we see the weakest trap observed at $\omega_y = 2\pi \times (0.211 \pm 0.004)$ Hz. Evidently the bias field deviation is smaller than in the old guide, otherwise we would expect the measured trap frequency in the new guide to be larger than the calculation.

### 3.3 Loading the Atoms

Loading the atoms into this trap proved to be a challenge. The loading sequence was complicated by two external fields, one from an environmental magnetic field and the other from gravity. With such weak magnetic confinement a small external bias field will shift the minimum of the trap along the guide. If the guide is not perpendicular to the
3.3. LOADING THE ATOMS

Figure 3.10: Loading the External Guide
(a)-(c) Show the position of the trap minimum, oscillation amplitude, and oscillation (trap) frequency respectively of the atoms throughout the final step of the wave-guide loading sequence. During this sequence the dc spherical quadrupole magnetic field is reduced to zero over 250 ms from a 0.7 Hz trap. (d) Shows the harmonic oscillation for the weakest trap observed, 0.211 ± 0.004 Hz. Here the atomic motion was purposefully excited to measure the frequency more accurately.
gravitational force, a small gravity component can also shift the overall trap minimum. The environmental magnetic field pointing along the guide can be canceled with a separate external \(y\)-bias field. Two coils of wire carrying small dc currents were placed co-axial with the guide for this purpose. The external guide can be manipulated to level the guide. Leveling the guide structure is not necessarily the same as making the weakest principle axis of the potential level. In practice the only way to cancel both the gravitational and magnetic external contributions is to minimize the atomic motion as the magnetic potential is weakened by reducing the spherical quadrupole magnetic gradient. It takes about 30 minutes to take enough data to discern the potential minimum and atomic motion so a perfect minimization takes a prohibitively long time to accomplish.

Fig 3.10(a)-(c) shows characteristic data for the dynamics of the atoms as they are loaded into the waveguide from the TOP trap after significant optimization of the guide tilt and \(y\)-bias field. In particular one can see how the position of the trap minimum migrates around due to a confluence of external magnetic and gravitational fields in (a).

The loading sequence is not adiabatic in the last step. This is because loading the atoms adiabatically into this trap takes long times of about 30 seconds, which is a significant fraction of the lifetime of the atoms in the new vacuum chamber (of the order of 2 minutes). Instead the atoms were loaded adiabatically into a trap with the spherical quadrupole still on and a \(y\) trap frequency of about 0.7 Hertz. Turning the trap suddenly down to 0.2 Hz produced breathing modes and trajectory oscillations of the atoms. The trap was ramped to 0.2 Hz in 250 ms because longer times did not seem to significantly reduce the dynamics while times shorter made the dynamics worse. The actual loading process is shown in Table 3.2
3.4 Electrical & Thermal Properties

The guide wires are made from Litz wire. Litz wire is a stranded cable but each strand is individually insulated from the others. At higher frequency both the impedance from the decreased skin depth and the proximity effect are reduced. At higher frequency, the current distribution in the cable is more uniform than a standard cable and the resistance is lower.

To determine the electrical characteristics of the guide wires we applied ac voltages to them from a similar current source as will be used in the experiment. The current through the wires is monitored using a TBH25P current sensor which is made up of a coil of wire forming in essence a transformer. Being careful to account for the effect of the current sensor and any hookup cabling, the resistance of the wires at dc using Ohm’s law is 0.15 \( \Omega \) for the bias fields and 0.20 \( \Omega \) for the linear quadrupole. The hookup cable resistance of 0.05 \( \Omega \) has already been accounted for in these values. One might think that the linear quadrupole should be twice the resistance of the bias fields since it has twice the length of wire, but the bias fields pass through a connector box that connects each turn of the circuit to the next turn and this box has significant resistance compared to the Litz wire itself.

The inductance was measured by monitoring the voltage, \( V \), and current, \( I \), amplitudes.

---

Table 3.2:
Sequence for loading atoms from the TOP trap into the Guide. Blank spaces indicate no change from the previous step was made.

<table>
<thead>
<tr>
<th>ramp time</th>
<th>ramp type</th>
<th>Spherical Quad. [Gauss/cm]</th>
<th>Linear Quad. [Gauss/cm]</th>
<th>Rotating Bias [Gauss]</th>
</tr>
</thead>
<tbody>
<tr>
<td>TOP trap</td>
<td></td>
<td>180</td>
<td>0</td>
<td>4</td>
</tr>
<tr>
<td>1 sec</td>
<td>linear</td>
<td>45</td>
<td>31</td>
<td></td>
</tr>
<tr>
<td>1 sec</td>
<td>linear</td>
<td>23</td>
<td>20</td>
<td></td>
</tr>
<tr>
<td>1 sec ( \tau )</td>
<td>exponential</td>
<td>2.3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>250 ms</td>
<td>linear</td>
<td>0</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
applied along with the phase, $\Upsilon$, between the current and voltage for some frequency, $\omega$. The impedance is $Z = V/I = \sqrt{R^2 + \chi^2}$ for resistance, $R$, and reactance $\chi = i\omega L$ in the case of negligible capacitance. Combining these relations with $\tan(\Upsilon) = R/\chi$ determines an equation for the inductance

$$L = \frac{V}{I\omega} \sin \Upsilon$$  \hspace{1cm} (3.9)

Values for the inductance were taken at frequencies from 1 to 20 kHz, while being careful to account for all the hookup wires. The inductance for the guide conductors was constant over these frequencies with values of $43.7 \pm 0.2 \mu H$ for each bias field and $44.5 \pm 0.3 \mu H$ for the linear quadrupole conductor.

The currents are sourced from the power supply for the old guide described in [58] modified slightly. The transformers and extra inductor are removed, and the capacitor is changed to $4 \mu F$ to make the $LC$ resonance of 11.9 kHz for the new inductance. The electrical diagram is shown in Figure 3.11. The removal of the transformers will require the power amplifier to source more current than it had to previously. The QSC Audio model RMX 1450 power amplifiers are built for commercial stereo audio use, but the two channels can be “bridged” to allow for the higher current needed for the new guide. The two bias fields are bridged in this way, but the linear quadrupole current is not because it is on for a sufficiently short time as to avoid heat overloads in the amplifier. The noise in the current is comparable to the previous waveguide (see Appendix E).

One negative consequence of the external guide is that when the dc magnetic trap is on while the ac fields of the guide are on (during evaporative cooling for example), the guide structure becomes a audio speaker at 11.9 kHz. This is mostly a nuisance to graduate students, but it is important that the capacitor above is not made from acoustically sensitive material such as ceramic; otherwise unwanted phase shifts between guide conductor circuits appear that are non-linear with applied current. Polypropylene seemed to be best.

For the 12 and 15 amps of ac amplitude needed in the bias fields and the quadrupole
field, we expect to dissipate about 40 Watts when the guide is fully loaded. Typically, these currents would be on for about 10 seconds. The heat capacity of Boron Nitride is 1610 J/kg-K and the density is 1.9 g/cm$^3$. The volume of the guide is 291 cm$^3$ taking into account every feature of the Boron Nitride structure except for the small mounting holes. This determines a mass of about half a kilogram. In the scenario of no heat being removed from the guide structure during the experiment, we could expect a temperature increase of half a degree Kelvin over the course of the experiment. The thermal expansion coefficient for Boron Nitride is $3.1 \times 10^{-6}$/K in the $x$ and $z$ directions and $11.9 \times 10^{-6}$/K in the $y$ directions.$^1$ The guide could expand by 80 nm in the $x$ direction, 73 nm in the $z$ direction and 1.5 $\mu$m in the $y$ direction. This corresponds to a $1.5 \times 10^{-6}$ relative difference decrease of the bias field magnitude and a $5.6 \times 10^{-4}$ relative difference decrease of the linear quadrupole gradient magnitude in the $x$ and $z$ directions. The $y$ direction is harder to predict, but an estimate could be made by assuming it depends upon the inverse square

Figure 3.11: New Guide Driver Circuit
(a) shows the circuit diagram for the new guide, modified from [58]. (b) is the preamplifier component of (a) shown in detail.

---

$^1$Properties of BN come from http://www.accuratus.com/boron.html which is the same company we acquired the BN from
3.4. ELECTRICAL & THERMAL PROPERTIES

As the guide heats up over the course of a day, the atoms seem to move slightly. This data was all taken with the Spherical Quadrupole Field on at 2.3 Gauss/cm.

of the length of the guide, which means the gradient would decrease by $1.2 \times 10^{-5}$ relative difference. The trap frequencies change by the square root of these values. These are all large over estimates because the guide is mounted to an effectively infinite heat reservoir in the optical table via two half-inch diameter by 2 inch long stainless steel posts. Conductive heat loss to the optical table combined with conductive and convective coupling to the air around the guide will decrease these changes. Also, divide the above values by ten if you would like to consider the expansion during a one second interferometer. The apparatus should be effectively unperturbed by thermal expansion in the guide.

The guide does heat by about 10 degrees Kelvin after several hours of continuous use. Some of this will be due to the air around the guide heating up as a result of the power dissipated by the cables connected to the spherical quadrupole field some distance away. The atoms do seem to move in response to the temperature as shown in Fig. 3.12. The analysis above indicates that it is unlikely that the atom position change is due to the BN guide structure. It could be as a result of the wires in the end connections moving around since the expansion coefficient of copper is much higher than BN. We do not observe fluctuations in the TOP trap position with temperature. This data is just a correlation to

Figure 3.12: Guide Temperature Effects
As the guide heats up over the course of a day, the atoms seem to move slightly. This data was all taken with the Spherical Quadrupole Field on at 2.3 Gauss/cm.
temperature; there could be some other parameter changing on long time scales that is the actual cause for the position drift. This drift makes optimizing the guide loading sequence more difficult, but otherwise should not affect the interferometer on the one second time scale.

3.5 External Guide Performance

The external wave-guide should be able to perform for measurement times five times longer than the internal guide because the trap frequency is five times smaller. Sadly, this performance is not achieved. The visibility of the interferometer in this new guide is shown in Fig. 3.13. The interferometer fails at about the same time as the internal guide. This means that the model presented in Chapter 2 is incomplete.

The decrease in visibility for the external guide looks qualitatively different from the internal guide in the two-reflect case and more like the four-reflect case in the internal guide. The visibility goes to zero in time, but not because $N_0/N \rightarrow 1/2$ independently of the applied standing wave phase $\alpha$ as in the single and two-reflect cases in the internal guide. Rather, $N_0/N$ becomes random and the average of $N_0/N \rightarrow 1/2$. Evidentially the phase is not washing out, but becoming random as time increases. For long enough times the spread in $N_0/N$ also decreases indicating a decrease in the underlying visibility. We can determine the time scale for this reduction by observing the variance in $N_0/N$ as we did for the free-oscillation case in Chapter 2. The variance $\Delta^2$ is related to the visibility $V$ by $\Delta^2 = V^2/8$. The results are shown in Figure 3.14 in comparison to the two and four reflect cases in the internal guide. One can see that were it not for the phase noise, the interferometer in the new guide could operate at twice the total time and maximum separation as the internal guide. The figure of merit, $\chi$ (see Chapter 2), could increase from 15 $\mu$m·s to 60 $\mu$m·s, but only if noise sources can be addressed.

There are many possible sources for noise, but there are two principle sources of noise
3.5. EXTERNAL GUIDE PERFORMANCE

Figure 3.13: External Guide Visibility

The external waveguide does not seem to be an improvement from the internal waveguide as in Fig. 2.7 of concern for this apparatus. One is laser phase noise. We’ve already established that changing the standing wave phase relative to the atoms will affect the phase the interferometer measures. We do this purposefully to apply a control phase to the interferometer for testing by changing the frequency of the laser between the split and recombination pulses. Clearly then laser frequency noise could be a problem. In addition to frequency, anything that affects the optical path length of the standing wave beam will affect the phase noise. Examples of path length altering phenomenon include movement of the standing wave mirror and changes to the index of refraction of the air between the mirror and the vacuum chamber.

To monitor laser phase noise a Michelson optical interferometer is used. Laser light for the standing wave is split with a beam splitter coming out of a fiber. The transmitted
3.5. EXTERNAL GUIDE PERFORMANCE

Figure 3.14: External Guide Variance
Shot-to-shot interference contrast fall off at slightly longer times than the 4-reflect case in the old waveguide, but at twice the maximum separation.

light forms the Bragg standing wave for the experiment. The reflected light from the beam splitter is reflected from a second mirror about 10 cm away to form a second standing wave. The reflected beams from the Bragg standing wave mirror and the secondary mirror recombine on the beam splitter and are sent both back into the fiber and onto a photodiode. Any phase noise between the two arms of the Michelson will show up on the photodiode. It should be pointed out that interference noise in the Michelson interferometer is not exactly the same as the noise the atom interferometer would experience, however the Michelson is nonetheless useful for indentifying noise sources. For example if one were to blow warm air from the mouth through one of the arms of the Michelson, the photodiode will show broadband full scale noise. To guard against stray air currents and temperature changes the beam paths are enclosed in thick paper tubes.

When both Michelson mirrors are rigidly mounted to the optical table, the photodiode shows that the optical interference pattern is stable for several seconds. This suggests that laser noise is not the primary source of atom-interferometer noise. One phenomenon
the Michelson interferometer cannot test for is the relative position of the standing wave mirror to the atoms. The atoms are an inertial reference that couple very weakly to the optical table through the external waveguide structure mounted to the table. The atoms could experience effects of the optical table moving where the Michelson would not because the Michelson is rigidly mounted to the table.

To test for this the Bragg beam mirror can be mounted to an inertially stabilized platform. We use a model Nano 20 by Halcyonics, which begins isolating frequencies higher than one Hertz and reaches its factor of 100 maximum isolation at 10 Hertz\(^2\). The platform uses geophones as sensors that feedback to PZTs to stabilize a square platform. In this configuration, the Michelson is sensitive to inertial movement of the table. It shows interference noise on the 100 ms time scale as a result indicating that the optical table is moving with frequencies of a few Hertz.

We can use geophones to measure the vibrations directly. Geophones work by detecting the induced voltage in a coil surrounding a magnetic mass on a spring. At frequencies below a few Hertz, the geophone ceases to work because friction in the device lowers the mass’s inertial response to motion. The geophone allows us to eliminate vibration noise sources at higher frequencies though. At one point, the interferometer was noisy even for short times and it was discovered that fast vibrations from a fan in an electronics shelf were coupling into the optical table. Isolating this electronics shelf from the table returned the interferometer to its usual performance, so vibrations are practically known to be a problem. The geophone can tell us that the passive vibration isolation of the table stops working around a 3 Hertz mechanical resonance. The interferometer becomes noisy for total times between 50 and 100 ms or 10 to 20 Hz. The geophone can detect linear vibration (shaking) of the optical table at these frequencies well, but amplitudes suggest that there is not enough shaking at these frequencies to cause the interferometer degradation we observe. The geophones cannot detect rotational vibrations (rocking) and

\(^2\)http://www.halcyonics.de/page/node/640
cannot detect low frequencies where we expect the vibration amplitudes to be the greatest.

Another approach was needed, one that could work to zero frequency and also one that could detect rocking motion because rocking will introduce phase noise from gravity. The solution was to shine a HeNe laser onto a four quadrant photodiode in such a way as to isolate rocking motion of the table. The difference signals between the top and bottom half or the left and right half can be monitored on an oscilloscope. The HeNe beam is centered on the diode so that both differences read zero. The 4 quadrant photodiode and the HeNe are mounted next to each other on the actively stabilized inertial platform to isolate them from building vibrations. The HeNe beam bounces off a mirror on the optical table 3 meters away. The beam plane is parallel with the longitudinal axis of the external wave guide, so that the rocking signals detected will be the principle ones experienced by the atoms, but not translations. The photodiode difference signal was calibrated with a translation stage. This apparatus measures 40 microradian rocking fluctuations at 1.3 and 3 Hz. We can deactivate the passive vibration isolation for the optical table by bleeding the air out of the air bladders it sits on. In this case the optical table is effectively rigid mounted to the floor. The quadrant photodiode measures 20 microradian fluctuations around 2 Hz as well as some higher frequency components. The passive support system damps out these higher frequency components of rocking motion at the cost higher amplitude rocking at the 3 Hz mechanical resonance.

The question is: Is this enough to explain any of the noise we observe in the interferometer? Here we will analyze the effect of a rocking motion on the interferometer. If the table is shaking at frequency $\Omega$ and amplitude $\theta$ in the small angle approximation then, there will be an addition to the potential energy from gravity $mgy\theta \sin[\Omega(t-t_0)]$, where $t_0$ accounts where in the oscillation the interferometer begin. The trajectory will be altered as a driven harmonic oscillator. After the split pulse the new trajectory is

$$y(t) = \left(\frac{v_0}{\omega} - \frac{g\theta \Omega}{\omega(\Omega^2 - \omega^2)}\right) \sin(\omega t) + \frac{g\theta}{\Omega^2 - \omega^2} \sin[\Omega(t - t_0)]$$

(3.10)
3.5. EXTERNAL GUIDE PERFORMANCE

The trajectory after the reflect pulses will be altered in the same way described in Chapter 2. There will be a resulting phase calculated by integrating over the energy altered by the gravity term. The result has the form \( \beta(\tau) \sin \Omega t_0 \), where again, \( t_0 \) is a random variable. To first non-zero order in \( \tau \):

\[
\beta \approx 16 k g \theta \left( \frac{\Omega^2 + \omega^2}{\Omega^2 - \omega^2} \right) \omega^2 \tau^4 \tag{3.11}
\]

Note the sensitivity of the phase to vibration frequencies equal to the trap frequency. To see what this random phase does to the visibility we must average over \( t_0 \)

\[
\left\langle \frac{N_0}{N} \right\rangle = \frac{1}{2} \left( 1 + V_0 \int_{-\infty}^{\infty} \cos \{ \phi + \beta(\tau) \sin[\Omega t_0] \} dt_0 \right) \tag{3.12}
\]

The substitution \( z = \sin \Omega t_0 \) along with a trigonometric identity rewrites the integral term as

\[
V_0 \cos \phi \int_{0}^{1} \frac{2 \cos[\beta(\tau)z]}{\pi \sqrt{1 - z^2}} dz \tag{3.13}
\]

where we’ve already removed the \( \sin \phi \sin \beta z \) integral which is zero. The integral evaluates to the Bessel function of zero order:

\[
\left\langle \frac{N_0}{N} \right\rangle = \frac{1}{2} \left( 1 + V_0 J_0[\beta(\tau)] \cos \phi \right) \tag{3.14}
\]

where we recognize the noisy visibility is

\[
V = V_0 |J_0[\beta(\tau)]| \tag{3.15}
\]

The absolute value acknowledges that a negative term will be interpreted by the fit as a shift in phase.

Before evaluating this expression, we should consider another effect of the oscillation. The atoms will recombine at a different location from where they split. There is a phase
associated with interacting with the standing wave beam at this different location of 
\( \alpha = k\Delta y \) as in Eq. (1.57) where \( \Delta y \) is the change in the two locations. This term is also 
proportional to \( \sin \Omega t_0 \) and so will also reduce the visibility via the Bessel function. The 
phase \( \alpha(t) \) to first non-zero order in time is

\[
\alpha \approx 2kg\theta\Omega^3 + 4kg\theta \left( \frac{\Omega^4 + \omega^4}{\Omega^2 - \omega^2} \right) \tau^4
\]

(3.16)

This term is more problematic, obviously. The resulting visibility loss for these random 
phases is shown in Figure 3.15. It shows that the visibility is predicted to fall off at slightly 
longer times than observed. This was done for tilting at one particular frequency. The 
visibility will fall off sooner if the whole spectrum of noise is taken into account.

One possibility to combat phase noise would be a differential measurement just as 
the group of M. Kasevich now does [60]. If after the split pulse, there were a “half-
reflect” pulse that purposely only reflected half the atoms: that is an operation 
\( |\pm 2\hbar k\rangle \mapsto \left( |2\hbar k\rangle + |-2\hbar k\rangle \right)/\sqrt{2} = |+\rangle \), which creates a superposition state, |+\rangle like the split operation 
does, but from the initial state | \( \pm 2\hbar k\rangle \) rather than |0\rangle. This operation would produce 
four packets from the split state \( (|2\hbar k\rangle + |-2\hbar k\rangle)/\sqrt{2} \). These four wave-packets will trace 
out the trajectory in Figure 3.16.

A differential measurement would have the advantage that common mode noise will 
cancel while a phase between one interfering pair relative to the other pair will not. Note 
that the output states have changed to four total states from three. The absorption signal 
to noise ratio will only decrease slightly. This idea will be discussed in more detail in 
Chapter 6.
The two curves show the predicted loss of visibility for table tilting at a frequency of 3 Hz with the interferometer starting at a random time during the tilt oscillation. The solid curve is for a tilting amplitude of 40 microradians, as the measurements indicate. The dashed curve is for 80 microradians for comparison. The data points are the visibility observed as in Fig. 3.13.
A half-reflect pulse splits the atoms a second time, now into two pairs (solid and dashed-dot curves). The four wave packets can freely oscillate until another half-reflect recombines them. There are four output states marked with dashes.

Figure 3.16: Differential Interferometer trajectory
Chapter 4

Rotation sensing

4.1 Introduction

Rotation sensing was one of the first recognized applications of atom interferometry and a catalyst of significant research [33, 60, 61]. The Sagnac effect [62] describes interference from rotation. Two waves on two paths that together enclose an area on a rotating platform (such as the Earth) interfere with a phase:

\[
\phi = \frac{4\pi \mathbf{A} \cdot \mathbf{\Omega}}{\lambda v}
\]  

(4.1)

where \( \mathbf{A} \) is the area enclosed with unit vector normal to the surface, \( \mathbf{\Omega} \) is the rotation psuedovector pointing parallel to the axis of rotation, \( \lambda \) and \( v \) are the wavelength and speed of the wave respectively. This phenomenon emerges in a ring cavity laser for example. The most sensitive laser gyroscope is the 367 square meter laser gyroscope in New Zealand [63]. Light travels in both clockwise and counterclockwise orientations around a cavity using the earth as a rotating platform, but light travelling with the earth’s rotation will be slightly blue shifted compared to light traveling against the rotation of the earth. The result is a beat note when light from these two directions are overlapped and directed at a photodiode.
For atoms, the wave is the quantum wave with corresponding deBroglie wavelength $\Lambda = h/p$ (Planck’s constant over the momentum). Equation (4.1) becomes:

$$\phi = 4\pi A \cdot \Omega \frac{m}{h}$$ (4.2)

and note the ratio of the atom wave Sagnac effect to the optical wave for the same area is

$$\frac{mc^2}{h\nu} = \frac{c}{v_r} \sim 10^{10}$$ (4.3)

Atoms are much more sensitive because their rest energy is so much higher than the photon energy. One could also think of the gain in terms of the slowness of the characteristic speed of atoms $v_r$ (5.85 mm/s for Rb using 780 nm light), compared to the speed of light, which provides more time for the Sagnac phase to develop. The 30 mm$^2$ area of the atom interferometer gyroscope in [61] is more sensitive by about an order of magnitude compared to the much larger laser gyroscope. The short term sensitivity is $6 \times 10^{-10}$ radians per second with a stability of about $2 \times 10^{-8}$ radians per hour.

The interferometer works by splitting an atomic beam traveling in free space into two slightly displaced paths with a series of laser beams as in Fig. 1.1. They follow a parallelogram shape like about two meters long and a few tens of micrometers wide. One limitation of this geometry is the inefficiency in enclosing a large area compared to the overall size of the device. This geometry may suffice for doing studies of the Earth’s rotation, but will limit other potential applications that require some degree of portability.

Several researchers realized that by using ultracold atoms supported against gravity, a more efficient, possibly more portable device could be created [55, 64, 65, 66, 67, 68, 69]. Atoms held against gravity, such as the case with the atom waveguide in our experiment, allow for longer interaction times than free space interferometers without increasing the size. The extra time could be used to make a large path or, with careful thought, pass several times around a loop before making a measurement thus increasing the effective
area of the loop without increasing the size of the device. To date only very small areas have been enclosed with this idea.

The device described here and in [70] constitutes one possible approach to making an ultracold atom gyroscope which, arguably, could find many more potential applications because it more easily expandable in area. The basic principle will be to take the linear interferometer described in previous chapters and add controlled motion perpendicular to the Bragg pulses. We establish perpendicular motion by inducing the atoms to harmonically oscillate in the $x$ direction with its relatively tight confinement (6.0 Hz), while the Bragg pulses operate in the $y$ direction (1.1 Hz) just as they did in the linear interferometer. Note, this work was done in the internal guide. Figure 4.2 shows the trajectory described.

This approach is most similar to that of Wu et al. [67]. In that work laser cooled Rb is loaded into a linear guide similar to ours, then that guide is physically translated perpendicular to a Bragg beam. The more significant difference is the use of laser cooled atoms as compared to the orders of magnitude colder Bose-Einstein condensate in our experiment. A moving guide with a BEC would be difficult because for the atoms to follow the guide motion, the guide would need strong fields and confinement. This tighter confinement creates a denser atomic sample and therefore increases the atom-atom interactions of the BEC as well as the confinement effects discussed in the Chapter 2.

On the other hand, using atoms that are warm compared to a BEC presents a challenge because the relatively broad momentum distribution makes the standing wave operations inefficient at transferring population to other momentum states. The Bragg operation will populate many momentum states and many possible paths for an atom to traverse during the experiment. In the end, the output is the interference pattern of all possible paths. As Wu demonstrated (see also [71]), it is possible to reconstruct a Sagnac signal, but questions remain as to how precise this technique can be. Also, if multiple loops are desired, then more pulses will likely be needed and the number of paths will grow
4.2 Extension to two dimensions

A Sagnac interferometer requires that the wave-packet trajectory enclose an area so the linear interferometer presented thus far must be extended to two dimensions. This can be accomplished by exciting transverse motion of the BEC in the waveguide and timing the longitudinal Bragg operations to the resulting transverse harmonic oscillation. We excite the transverse harmonic oscillation by manipulating the phase of the current for the oscillating linear quadrupole magnetic field relative to the field of the rotating bias field. When we operate the interferometer in a linear geometry, the combined ac magnetic fields create a line of zero field that moves in a horizontal plane above the atoms. Adjusting the relative phase between the two fields effectively tilts the plane by a corresponding angle about the $y$ axis. The trap minimum of the time averaged field will shift in the $x$ direction, as well as $z$.

One might like to make sinusoidal changes to the phase to drive transverse motion, but this was not possible with the technology available. We make this phase shift suddenly instead. After the shift the atoms find themselves on a potential hill and will move towards
the center and begin to oscillate harmonically with the distance of the trap shift as their amplitude. Generally, this sudden shift of the potential produces oscillations in all three directions. If, however, the trap is shifted back to the original location at a suitable time later, the $x$ oscillations can be enhanced while the other directions are reduced.

One can solve the classical problem of a harmonic oscillator with a square wave driving force to understand this situation and figure out when to shift the trap and how many times the trap needs to be shifted. Let the square wave be represented as a sum of pulses

$$X_{sw}(t) = \sum_{n=0}^{N-1} \begin{cases} 1 & n\tau < t < (n+1)\tau/2 \\ 0 & \text{else} \end{cases}$$

(4.4)

where $\tau$ is the period of the square wave and $N$ is the number of pulses. The equation of motion is just,

$$\ddot{x}_i + \omega_i^2 x_i = A_i \omega_i^2 X_{sw}(t)$$

(4.5)

where $A_i$ is the amplitude of the square wave. The solution is easiest with Laplace transforms defined as

$$L\{f(t)\} = F(s) = \int_0^\infty e^{-st} f(t) dt$$

(4.6)

$$L^{-1}\{F(s)\} = f(t) = \frac{1}{2\pi i} \int_{\gamma-i\infty}^{\gamma+i\infty} e^{st} F(s) ds$$

(4.7)

where $\gamma$ is real and exceeds the real part of the singularities in $F(s)$. The transforms and properties needed can be looked up in standard math tables [72].

The equation of motion becomes

$$s^2 F_i + \omega_i^2 F_i = \omega_i^2 A_i \frac{1 - e^{-s\tau/2}}{s(1 - e^{-s\tau})}$$

(4.8)
Figure 4.1: The upper plot shows how a square wave with two periods of the indicated length of time will amplify the oscillation amplitude of a harmonic oscillator as a multiple of the amplitude of the square wave, $A_i$. The lower plot blows up the time scale around 1.95 seconds and the legends refer to the directions of the trap where $x$ is the desired oscillation direction.
where a property of Laplace transforms for periodic functions, \( f(t + \tau) = f(t) \),

\[
F(s) = \frac{\int_0^\tau e^{-st} f(t) dt}{1 - e^{-s\tau}} \tag{4.9}
\]

was used. Rearranging Eq. (4.8) we find

\[
F_i = \frac{\omega_i^2 A_i}{s^2 + \omega_i^2} \frac{1}{G_i \left(\frac{s(1 + e^{-s\tau/2})}{H}\right)} \tag{4.10}
\]

Laplace transforms begin to show their utility as we can now use a property of the inverse transform dealing with products of functions and their convolution:

\[
L^{-1}\{G(s)H(s)\} = (g * h)(t) = \int_0^t g(t - \nu)h(\nu) d\nu \tag{4.11}
\]

where we recognize that \( g(t) = A_i \omega_i \sin \omega_i t \) and \( h(t) = X_{sw}(t) \) above, which is periodic.

From here we can find \( x_i(t) \). What we really want is \( x_i(N\tau) \), the position (and velocity) after the square wave runs \( N \) periods. The motion will be simple harmonic after this. The solution is then

\[
x_i(N\tau) = A_i \omega_i \sum_{n=0}^{N-1} \int_{n\tau}^{(n+1/2)\tau} \sin [\omega_i (N\tau - \nu)] d\nu \tag{4.12}
\]

\[
= 2A_i \sum_{n=0}^{N-1} \sin \left[\frac{\omega_i \tau}{4}\right] \sin \left[\omega_i (N - n - 1/4)\right] \tag{4.13}
\]

\[
= 2A_i \frac{\sin \left[\frac{\omega_i \tau}{4}\right]}{\sin \left[\frac{\omega_i \tau}{2}\right]} \sin \left[\frac{N\omega_i \tau}{2}\right] \sin \left[\omega_i (N/2 + 1/4)\right] \tag{4.14}
\]

using a combination of trigonometric identities. Similarly one can go through this analysis and find the velocity as

\[
v_i(N\tau) = 2A_i \omega_i \frac{\sin \left[\frac{\omega_i \tau}{4}\right]}{\sin \left[\frac{\omega_i \tau}{2}\right]} \sin \left[\frac{N\omega_i \tau}{2}\right] \cos \left[\omega_i (N/2 + 1/4)\right] \tag{4.15}
\]
From here, one can take these as the initial conditions for an oscillator and find what the trajectory should look like.

In Figure 4.1 we show the above result calculated for \( N = 2 \) and the trap frequencies observed. From this information we can note several features. One is that the tighter the confinement in a given direction, the more difficult it is to drive oscillations in that direction. This means that we need to be as careful as possible is selecting where to operate. The peak at 1.95 seconds is conveniently located at the minimum of both the \( y \) and \( z \) responses and it as this period, where the Sagnac interferometer will be operated.

### 4.3 Sagnac interferometer

Using the method previously described, we excite motion with observed amplitudes from \( C_x = 400 \) \( \mu \)m to 1.3 mm. We split the atoms in the \( y \) direction into the \( p_y = \pm 2\hbar k \) momentum states moving at \( v_0 = 11.7 \) mm/s just as in the linear case. This using a standing wave at \( \lambda = 780.1 \) nm, which is 70 GHz blue of the \( 5S_{1/2} \leftrightarrow 5P_{3/2} \) and has a diameter of 1 cm. This large standing wave is key since it allows for a uniform intensity at the \( x \) position of both the split and reflect pulses. To accomplish this larger detuning and larger beam we used a Sacher Tiger diode laser capable of producing \( \approx 1 \) Watt of light at these wavelengths. This laser has poor frequency stability compared to other diode lasers available, but at these large detunings the 10 MHz or so of frequency jitter as measured on a Coherent scanning Fabry-Perot spectrum analyzer is not a problem.

We symmetrize the pulse sequence about the \( x \) turning point. Therefore the two reflect interferometer has the split pulse at time \( t = 0 \), the reflect at \( t = \tau \), the turning point at \( t = 2\tau \) (here the atoms pass through each other), the second reflect at \( t = 3\tau \), and recombination occurs at time \( t = 4\tau \). Refer to Figure 4.2 to see the shape this two dimensional interferometer takes.

As with the linear interferometer previously described, we test the interferometer by
shifting the phase of the standing wave between splitting and recombination to apply a phase and then monitor the subsequent change to the relative populations of atoms in the $|0\rangle$ and $|\pm 2\hbar k\rangle$ momentum states, where now we are only labeling the $y$ momentum, not the total. All states should have the same $x$ component of momentum. The signal is $N_0/N = (1 + V \cos \phi)/2$ for visibility $V$, just as before. Any extra phase, including the Sagnac effect, will be in addition to the applied phase and appear as an offset. This experiment was done in the original vacuum waveguide so the experiment was limited to $4\tau = 72$ ms total times due to the phase gradients as well as possibly table and building vibrations mentioned previously. The time was constrained to $4\tau = 40$ ms in this experiment. This is because at these times the roll off of the 0.9 peak visibility in the linear interferometer has yet to occur so any reduction to the visibility should be due to
new effects. The shape shown in Fig. 4.2(c) has a physical area of

\[ A \approx \frac{4\nu_0 C_x}{\omega_x} (2 \sin \omega_x \tau - \sin 2\omega_x \tau) \]  

(4.16)
as long as \( \omega_y \tau \ll 1 \). The area used in the Sagnac phase would be twice this since both atomic wave packets traverse the full loop. Several example interferometer curves are shown in Fig. 4.3.

The Sagnac interferometer was phase stable only for small areas less than 0.2 mm\(^2\).
Here the offset phase is plotted vs the Sagnac area, which is twice that in equation 4.16 and the expected phase from the Sagnac effect due to Earth’s rotation. We see that any Sagnac phase is swamped by other phase sources.

For larger areas the phase noise grows large and reduces the visibility, even as shot-to-shot visibility remained high. This is similar to the linear interferometer at long times shown in Chapter 3. The sensitivity of the device is $\frac{\Omega}{\delta \phi} = 2$ millirad/s per radian of interferometer sensitivity $\delta \phi$. We find $\delta \phi$ of about 10 to 100 milliradian. Because we do not have access to a rotation platform, the only Sagnac phase we could potentially measure with this device is that of the Earth: 73 $\mu$rad/s. Compared to the signal noise the expected contribution to the phase from the rotation of the Earth is far too small to be observed here. See Figure 4.4. There is some other phase(s) affecting the interferometer that need to be determined.

One clue in the data is an asymmetry in the output state. As previously discussed the output states expected are $|0\rangle$ and $|+2k\rangle + |-2k\rangle$ where normally the second state has a symmetric population density in both moving components. In this experiment

Figure 4.4: Area-enclosing interferometer phase measurement

This is similar to the linear interferometer at long times shown in Chapter 3. The sensitivity of the device is $\frac{\Omega}{\delta \phi} = 2$ millirad/s per radian of interferometer sensitivity $\delta \phi$. We find $\delta \phi$ of about 10 to 100 milliradian. Because we do not have access to a rotation platform, the only Sagnac phase we could potentially measure with this device is that of the Earth: 73 $\mu$rad/s. Compared to the signal noise the expected contribution to the phase from the rotation of the Earth is far too small to be observed here. See Figure 4.4. There is some other phase(s) affecting the interferometer that need to be determined.
Figure 4.5: Area-enclosing Interferometer Asymmetry
Here the ratio of the atoms in the two moving states $|\pm 2\hbar k\rangle$ is plotted versus applied phase. This ratio oscillates at twice the period of the interference signal $N_0/N$.

however, this output state exhibited an asymmetry that depended on the applied phase, but at twice the period of the interferometer curves in Figure 4.3. An example is shown Figure 4.5.

Another clue is that although the phase noise lowers the visibility at larger areas, the visibility is not well correlated with area. Instead, the best correlation to the fall off in visibility seems to be with the speed at the time of the split pulse. The visibility for all six combinations of time and $x$ amplitude is shown in 4.6 as a function of this speed, which can be calculated as $v_{\text{split}} = C_x\omega_x \sin \omega_x \tau$.

Any effect of $v_x$ should in principle cancel since it is common to both packets. There are however subtleties that break this symmetry. First, there is an amplitude in the $y$ direction of about 0.1 mm/s and it is possible that this amplitude grows as we increase the $x$ amplitude. As discussed in Chapter 2 this motion is potentially detrimental to the interferometer. To counter this effect, the Sagnac interferometer was operated as near as possible to a turning point of the $y$ motion. The residual motion could be measured, and
4.3. SAGNAC INTERFEROMETER

Figure 4.6: Area-enclosing interferometer visibility
The visibility of the interference is shown versus the speed in the transverse direction at the time of the split pulse. The open circles are for 20 ms times and the filled circles for 40 ms total time. The labels denote the area enclosed in mm².

is too small to explain the limit we observe.

A more serious problem comes from a misalignment of the Bragg beam to the principle axes of the guide. Observations of the trajectory suggest a misalignment of a few degrees in the horizontal $x$ direction and about six degrees in the vertical direction. This misalignment is difficult to correct because the optical access to the waveguide was limited for the large area beam needed. In the new guide this will not be a problem, and once the table motion is reduced, another attempt at a Sagnac interferometer may prove fruitful.

Due to the misalignment, the Bragg standing wave will sample the projection of atomic motion in the $x$ and $z$ directions, which depends on $v_x$. This produces an effective speed in the $y$ direction of about 0.2-1 mm/s. Referring to Fig. 1.6, one can see the effect of a 5% to 10% change in the velocity from $v_0$ on the fidelity of the reflect pulse particularly. As the pulse fidelity falls, the standing wave begins to populate unwanted momentum states. These extra momentum states can traverse different loops and interfere with the primary
wave packets of the interferometer in a complicated way that will be the subject of the next section.

The phase between the primary states is also affected because the packets will traverse the loop with slightly different kinetic energies and thus makes them non-reciprocal. Consider the classical action in this trap

\[ \Phi = \frac{S}{\hbar} = \frac{m}{2\hbar} \int dt (|v|^2 - |q|^2) \]  \hspace{1cm} (4.17)

where \( v \) is the velocity and \( q_i = \omega_i x_i \). This integral can be carried out for the trajectory described earlier and evaluates to

\[ \Phi = \frac{mv_0}{\hbar} \hat{b} \cdot c \]  \hspace{1cm} (4.18)

where \( \hat{b} \) is a unit vector pointing along the Bragg beam and \( c \) is a vector whose components are \( c_i = C_i f(\omega_i \tau) \) for oscillation amplitudes \( C_i \) and

\[ f(\omega \tau) = \cos(6\omega \tau) - 2 \cos(5\omega \tau) + 2 \cos(3\omega \tau) - \cos(2\omega \tau) \]  \hspace{1cm} (4.19)

If \( \omega \tau \ll 1 \), \( f \to 8(\omega \tau)^4 \).

As an example calculation, suppose we use \( \tau = 10 \text{ ms} \), alignment errors of one degree in the horizontal and six degrees in the vertical, an \( x \) amplitude of one millimeter and no amplitude in the other directions. Then we would get a motional phase error of \( \Phi = 12 \text{ rad} \). This is similar to Eq. (2.38) in Chapter 2. First, this would be a significant error for any rotation measurement, but additionally if \( C_x \) fluctuates with a standard deviation \( \sigma = sC_x \), then the visibility will be reduced by a factor

\[ V = V_0 e^{-s^2 \Phi^2/2} \]  \hspace{1cm} (4.20)

From measuring the \( x \) oscillation and fitting it to a sine wave we can deduce that there
are $\approx 5\%$ fluctuations is the amplitude. This corresponds to a 60\% reduction in visibility.

This is in qualitative agreement with the $\tau = 10$ ms data, but the $\tau = 5$ ms data will have a motional phase 16 times smaller and we therefore expect much higher visibility for the same velocity error. We do not observe this. To press further, we will have to model the effect of the extra states caused by the velocity sensitivity of the Bragg operations.

### 4.4 Modeling a 3d interferometer

In an attempt to better understand how this interferometer should work, it is instructive to make a full quantum model. To do this we split the problem into two pieces. One piece is understanding the center of mass dynamics of a BEC in a harmonic trap. We’ll use the coherent state basis here. The other is to understand how the BEC interacts with the Bragg standing wave and for this we will use plane wave momentum states because the momentum spread is so narrow. This basis is the same as used to calculate the fidelities of the operations in Chapter 1. The pulses are on a very short time compared to the harmonic dynamics so we should be able to ignore dynamics when considering the pulses. In particular we need to keep track of the phases for the entirety of the experiment as well as their position and momentum information. These are all tied together of course since the phase depends on the energy and the energy depends on the momentum and position within the potential.

Consider the coherent state defined as

\[ |\alpha\rangle = D(\alpha)|0\rangle \quad (4.21) \]

where

\[ D(\alpha) = e^{\alpha \hat{a}^\dagger - \alpha^* \hat{a}} \quad (4.22) \]

and $|0\rangle$ is the harmonic oscillator ground state $(2/\pi x_0^2)^{1/4}\exp(-x^2/x_0^2)$. We are using the
usual ladder operators for the harmonic oscillator: \( \hat{a} = \hat{x}/x_0 + i\hat{p}/p_0 \). Also, \( x_0 = \sqrt{2\hbar/m\omega} \), \( p_0 = \sqrt{2m\hbar\omega} \) and the complex number \( \alpha \) contains the expectation values of the position and momentum, \( \bar{x} \) and \( \bar{p} \) respectively, via \( \alpha = \bar{x}/x_0 + i\bar{p}/p_0 \).

We also need the plane wave states.

\[
|p_n\rangle = \frac{1}{\sqrt{\hbar}} e^{ip_n x/\hbar}
\]  

(4.23)

where the index \( n \) will label the initial \( y \) momentum states \( p_{n,y} = n2\hbar k \) produced by the Bragg operation. Initial momentum will also be taken into account within \( p \).

With this knowledge we can create a basic model. The plan of attack will be as follows: One, numerically calculate given some initial conditions the distribution of atoms among the momentum states \( |p_n\rangle \) resulting from the Bragg operation. Two, project these states onto coherent states and propagate in the harmonic trap for some time. Three, project back to the momentum states recording whatever phase was accumulated during the propagation and applying the next pulse. Four, repeat as needed. We will need to find the projection of the coherent state basis on the plane wave basis to move back and forth so we first need to calculate the coefficients.

\[
B_p = \langle p|\alpha\rangle = \langle p|D(\alpha)|0\rangle
\]  

(4.24)

Using the identities above, one can show that

\[
D(\alpha) = e^{i(p\hat{x} - \hat{x}p)/\hbar}
\]  

(4.25)

Then one can use the fact that the commutator \( [\hat{x},\hat{p}] = i\hbar \) and the identity \( e^{A+B} = e^A e^B e^{-[A,B]/2} \) to show that

\[
D(\alpha) = e^{i\hat{p}\hat{x}/\hbar} e^{-i\hat{x}\hat{p}/\hbar} e^{-i\hat{p}\hat{x}/2\hbar}
\]  

(4.26)
From here, examine

\[
\langle x | D(\alpha) | p \rangle = e^{-i \bar{x} \bar{p}/2h} \langle x | e^{i \bar{p} \hat{x} / \hbar} e^{-i \bar{x} \hat{p} / \hbar} | p \rangle = e^{-i \bar{x} \bar{p}/2h} \langle x | e^{i \bar{p} \hat{x} / \hbar} e^{-i \bar{x} \hat{p} / \hbar} | p \rangle \tag{4.27}
\]

Now that the operators \( \hat{x} \) and \( \hat{p} \) are used, we can commute the two exponentials on the inside and incorporate the \( e^{i \bar{p} \hat{x} / \hbar} \) into \( |p\rangle \) to see that (4.27) becomes

\[
e^{-i \bar{x} \bar{p}/2h} \langle x | e^{-i \bar{x} \hat{p} / \hbar} | p + \bar{p} \rangle = e^{-i \bar{x} \bar{p}/2h} e^{-i \bar{x} \hat{p} / \hbar} \langle x | | p + \bar{p} \rangle \tag{4.28}
\]

So the operator \( D(\alpha) \) alters the plane wave state with the expectation value of momentum \( \bar{p} \) and gives it a phase term \( \bar{p} \bar{x}/2h \) along with another piece that modulates the amplitude of the state depending on the position. This was independent of what \( |x\rangle \) is, so we can generalize to:

\[
D(\alpha) | p \rangle = e^{-i \bar{x} \bar{p}/2h} e^{-i \bar{x} \hat{p} / \hbar} | p + \bar{p} \rangle \tag{4.29}
\]

This allows us to calculate the coefficients, \( B_p \) from Eq. (4.24).

\[
B_p = \langle p | D(\alpha) | 0 \rangle = e^{-i \bar{x} \bar{p}/2h} \langle p + \bar{p} | 0 \rangle \\
= \left( \frac{2}{\pi \hbar^2 x_0^2} \right)^{1/4} e^{-i \bar{x} \bar{p}/2h} e^{-i \bar{x} \hat{p} / \hbar} \int_{-\infty}^{\infty} e^{-i (p + \bar{p}) x / \hbar} e^{-x^2 / x_0^2} dx \\
= e^{-i \bar{x} \bar{p}/2h} e^{-i \bar{x} \hat{p} / \hbar} \left( \frac{2}{\pi p_0^2} \right)^{1/4} e^{-(p + \bar{p})^2 / p_0^2} \tag{4.30}
\]
Now we can project back and forth between the \(|\alpha\rangle\) basis and the plane wave basis with

\[
|\alpha\rangle = \int |p\rangle \langle p| \alpha \rangle dp = \int |p\rangle B_p dp
= e^{i\bar{x}\bar{p}/2\hbar} e^{-i\bar{p}\bar{x}/\hbar} \left( \frac{2}{\pi \bar{x}_0^2} \right)^{1/4} e^{-\frac{(x-\bar{x})^2}{\bar{x}_0^2}}
= e^{i\bar{x}\bar{p}/2\hbar} e^{-i\bar{p}\bar{x}/\hbar} |0(\bar{x})\rangle
\]

(4.31)

where \(|0(\bar{x})\rangle\) is the harmonic oscillator ground state centered at \(\bar{x}\). This says that the coherent state projection from a plane wave state is a the harmonic oscillator ground state centered at \(\bar{x}\) modified by a phase that is the product of the momentum and position, \(\bar{x}\bar{p}\), and another phase that depends on position \(x\) within the state. We could use this to deal with phase gradients as in Chapter 2, but we know from the data that phase gradients are not the problem so will only treat the overall phase \(\bar{x}\bar{p}/2\hbar\). The state \(|\alpha\rangle\) is evolved in time as \(|\alpha(t)\rangle = |e^{-i\omega t}\alpha(0)\rangle\) but all this says is that the momentum and position change harmonically i.e. \(\bar{x}(t) = x_0 \text{Re}[\alpha(t)]\) and \(\bar{p}(t) = p_0 \text{Im}[\alpha(t)]\).

For the interferometer, the Bragg operation will produce plane wave states with a range of amplitudes, of which we need only consider the first few orders. These plane wave states can be projected onto a coherent state with Eq. (4.31), which involves a phase term \(e^{i\bar{x}\bar{p}/2\hbar}\). The coherent state evolves in time until the next Bragg operation. The coherent state can be projected back into plane wave states, but with altered momentum and phases. This next Bragg operation manipulates the relative amplitudes of the states, and then we can start over again.

In order to make this three dimensional, recall that both the coherent states and the plane wave states are separable so we can treat each dimension individually. The Bragg beam is not aligned to one of the principle axes of the harmonic potential so generally will manipulate the momentum in all directions. This can be accounted for by using a rotation operation when transforming between the plane wave and coherent state bases. Let the prime coordinate system be aligned with the Bragg beam and the non-prime coordinate
system be for the harmonic oscillator, i.e the Bragg beam produces a state $|p'_n\rangle$ in the $y'$ direction with amplitude $c_n$. Then the same plane wave in the harmonic oscillator coordinates is $|p\rangle = R_\theta R_\phi |p'\rangle$ for rotation operators $R$ with angle $\phi$ about $x$ and $\theta$ about $z$. In matrix form:

$$
\begin{pmatrix}
-\sin \phi \\
\cos \theta \cos \phi \\
\sin \theta \cos \phi
\end{pmatrix}
= 
\begin{pmatrix}
1 & 0 & 0 \\
0 & \cos \theta & -\sin \theta \\
0 & \sin \theta & \cos \theta
\end{pmatrix}
\begin{pmatrix}
\cos \phi & -\sin \phi & 0 \\
\cos \phi & \sin \phi & 0 \\
0 & 0 & 1
\end{pmatrix}
\begin{pmatrix}
c \\
0 \\
0
\end{pmatrix},
$$

(4.32)

This defines $p'_j$ in terms of the rotation coefficients above and $p_j$. We can now walk through the steps listed above. Just after the split pulse we have a state $|\psi\rangle$ that is a superposition of plane wave states with amplitudes $c_n$ in the prime frame of the Bragg beam

$$
|\psi\rangle = \sum_n c_n \prod_j |p'_j, n\rangle
$$

(4.33)

where $j$ is indexing the directions $x$, $y$, and $z$. Rotating and projecting $|\psi\rangle$ onto the coherent states and propagating it in time produces:

$$
|\psi\rangle = \sum_n c_n \prod_j e^{i\xi_{j,n}/2\hbar} e^{-i\omega_j t} \alpha_{j,n}(\vec{x}'_{j,n}, \vec{p}'_{j,n})
$$

(4.34)

Projecting back to the plane wave states in the prime frame yields

$$
|\psi\rangle = \sum_n c_n \prod_j e^{i[\vec{x}'_{j,n}(0)\vec{p}'_{j,n}(0) - \vec{x}'_{j,n}(t)\vec{p}'_{j,n}(t)]/2\hbar} e^{-i\omega_j t} \alpha_{j,n}(\vec{x}'_{j,n}, \vec{p}'_{j,n})
$$

(4.35)

Here let us define

$$
\xi'_{j,n} = \vec{x}'_{j,n}(0)\vec{p}'_{j,n}(0) - \vec{x}'_{j,n}(t)\vec{p}'_{j,n}(t)
$$

(4.36)

The phase $\xi'_{j,n}$ represents the phase accumulated by the wave packet initially diffracted into the $n^{th}$ order by the split operation due to its propagation in the $j^{th}$ direction of
4.4. MODELING A 3D INTERFEROMETER

the harmonic trap. The state $|\psi\rangle$ will diffract from another Bragg pulse producing a new set of momentum states we will label with an $m$. These states will propagate and pick an additional phase $\xi_{j,n,m}$ similar to the $\xi_{j,n}$. This procedure continues for second reflect pulse and eventually the final recombination pulse, picking up $\chi_{j,n,m,q}$ and $\gamma_{j,n,m,q,r}$ phases. Just to re-iterate: $\chi_{j,n,m,q}$ is the phase accumulated from propagating in the $j^{th}$ direction in the harmonic trap by the state that was split into the $n^{th}$ order momentum state, then reflected into the $m^{th}$ order, then reflected into the $q^{th}$ order by the second reflect operation. The final state after the interferometer sequence is

$$
|\psi\rangle = \sum_{n,m,q,r} c_{n,m,q,r} \prod_j \exp\left[i/2\hbar (\xi'_{j,n} + \zeta'_{j,n,m} + \chi'_{j,n,m,q} + \gamma'_{j,n,m,q,r})\right] |p'_{j,n,m,q,r}\rangle \quad (4.37)
$$

As an example, let us look at the total phase accumulated by one of the principle inputs to the recombination operation: the plane wave state $|p'_{y,1,-1,1}\rangle$ in the longitudinal direction, $y$. It had momentum $p_y = 2\hbar k$ after the split, then $p_y = -2\hbar k$ after the first reflect, then $p_y = 2\hbar k$ after the second reflect. The total phase, $\Phi_{1,-1,1}$, for this state is

$$
\Phi_{1,-1,1} = \frac{1}{2\hbar} \sum_j \xi_{j,1} + \zeta_{j,1,-1} + \chi_{j,1,-1,1} \quad (4.38)
$$

The sum on $j$ comes from the product of the exponentials in (4.37). One can rewrite $\Phi_{1,-1,1}$ in terms of position and momentum information at the times of the pulses from Eq. (4.36).

In principle, the sum over diffracted orders would be infinite, but in practice there is a vanishing wave-packet amplitude in states of momentum higher that $4\hbar k$ so the sum will be truncated at $n, m, q, r = \pm 2$. This leaves $5^4 = 625$ possible trajectories to keep track of. Only trajectories whose final position overlaps with the two principle trajectories, $c_{1,-1,1}$ at the time of the recombination pulse will lead to interference effects that contribute to the overall signal of the interferometer and only these will be considered. It is helpful to think of the extra paths in terms of the order of error from the intended operation. The
reflect operation ideally takes atoms in the \( n = 1 \) state and transfers them to the \( m = -1 \) state so the small amplitude in the \( m = 0 \) or \( m = 1 \) states would be first order errors. The second reflect should not change the first order error \( m = 0 \) state amplitude so atoms transferred to the \( q = -1 \) state from that \( m = 0 \) state would be second order in smallness of amplitude. Several error trajectories are shown in Figure 4.7.

Referring to Figure 4.7, there is only one first order error: the case that the split pulse produces some amplitude of atoms that continue in the rest state. There are six second order paths. One potential complication is that there is another interference event at the second reflect pulse where three distinct paths can combine. There is an overlap of atoms on the primary path, a first order error (zero state atoms from the first reflect) as well as a possible (but very low probability) second order error path if the split pulse produces some population in the \( p = \pm 4\hbar k \) states, that are then placed in \( \pm 2 \) states by the first reflect. The second reflection is therefore sensitive to noise phases, which in turn adds to the sensitivity of the final recombination.

A program attached in Appendix C was written in Matlab to calculate the various plane wave amplitudes, \( c_{n,m,q,r} \) for the paths described above. The input parameters were: the position of the initial cloud relative to the potential, the momentum, two angles describing the misalignment of the Bragg beam to the principle axes of the potential, a noise parameter characterizing amplitude fluctuations (as we did with \( s \) in Equation (4.20)), and an applied phase, \( \alpha \), from purposely changing the frequency of the standing wave in the recombination operation. Note the initial position and momentum can equivalently be described by the three oscillation amplitudes along with a temporal offset to describe where in the oscillation the interferometer is starting.

From these input parameters, a set of four random amplitudes is chosen from a normal distribution of standard deviation \( s \) centered of the oscillation amplitudes used in the experiment. The ratios \( N_0/N \), the asymmetry \( N_+/N_- \), and any other observable desired are calculated. The applied phase \( \alpha \) is varied over about \( 4\pi \) and a new set of oscillation
Figure 4.7: Trajectory Errors
Schematically shown are atom trajectories that contribute to the measured output up and including 2nd order errors. The principle trajectories are shown in bold solid lines. First order errors are dashed lines and second orders are dotted lines. Numbered labels denote Bragg diffraction orders and the box contains a list of labels for the trajectories that contribute to the measured output interference pattern. The circles denote events that will show inference effects. Also shown, in gray, are example trajectories that do not contribute to the output.
4.5 3D MODEL RESULTS

In the lab we measured the following relevant parameters for the simulation. The angles of the Bragg beam to the principle axes of the guide were estimated to be 6° from the vertical and 1° in the horizontal. From a fit to the $x$ oscillation data we could determine that the fluctuations on the amplitude ranged between 3% and 5% of the amplitude. From the same data we could determine the $y$ motion was about 100 to 200 $\mu$m/s.

In Figure 4.8 one can see visually the effect of the errors presented on the actual paths the interferometer wave-packets take. The atoms start on the left at $y = 0$, proceed...
Figure 4.9: Simulated Moving State Asymmetry
Here one can see that the simulated asymmetry in the moving states, $|\pm 2\hbar k\rangle$ (open circles) compared to the observed asymmetry. The applied phase axes is derived by fitting the observed $N_0/N$ data and setting the period of oscillation to $2\pi$. The data is for an $x$ oscillation amplitude of 400 $\mu$m with fluctuations of 5%, total interferometer time of 40 ms, 200 $\mu$m/s of $y$ motion, and offset angles of 1° and 6°.

around the loop and finish at slightly positive $y$. First, the paths lose their reciprocity so there will not be as good a cancelation effect for any asymmetries in the guiding potential. Also, the atoms do not precisely overlap at the end. The program does not account for imperfect packet overlap, which should be small on the scale of the packet size, nor does it account for the phase gradient effects, or anharmonic effects described in Chapter 2, which should by design be small for the interferometer times presented in this chapter.

One key test of the simulation is that it can predict the asymmetry in the moving state
4.6. Sagnac Conclusion

To conclude this section on a Sagnac interferometer, we realize that for future progress to be made a combination of improvements are needed. One important improvement will be to stabilize the transverse oscillations. The fluctuations are thought to result from poor timing stability of the magnetic trap shifts. The shifts were controlled via a GPIB computer interface. The timing resolution of the GPIB is on the millisecond time scale for a 2 meter cable, but it’s unclear what the timing resolution of the Agilent 33220A function
Figure 4.10: Simulated Sagnac Visibility
Here the simulated visibility is plotted against the speed of the atoms at the time of the split pulse just as in 4.6 and for the same oscillation amplitudes and times. The data is for an $x$ oscillation fluctuations of 5%, 200 $\mu$m/s of $y$ motion, and offset angles of 1° and 6°.

Another improvement could come from an alternative guide geometry, one that allows the split pulse to happen at an $x$ turning point. In that case, there will be no residual motion for the Bragg beam to sample even if it were slightly misaligned, and in a new apparatus care could be taken to ensure that optical access will allow correction for most misalignment problems. To force the split to happen at a turning point, a new guide will need to be either weaker in the $y$ direction to allow for longer times, or stronger in
the $x$ direction to increase the trap frequency. We have previously attempted to run in the trap with tighter $x$ confinement (see Figure 2.8) and observed a loss of interferometer performance so of the two options, weaker $y$ confinement appears to be the better option. This experiment has not been repeated in the new guide of Chapter 3 yet. In the new, flatter guide, the phase gradient problem has been reduced such that longer times and $y$ displacements are available, but as we saw, more phase noise is introduced. The variance data displayed in Fig. 3.14 demonstrate that the new guide should be able to support times up to 160 ms. The time needed to split at an $x$ turning point is about 170 ms. The phase noise problems must be dealt with to see if phase stable operation at these times will be possible. If these times were possible, the Sagnac area enclosed for an $x$ oscillation amplitude of 1 mm, would be 2.5 mm$^2$ for one loop and would have a sensitivity $\Omega/\delta\phi = 140 \mu$rad/s per radian of phase sensitivity. This gyroscope would be sensitive to Earth’s rotation. This geometry could be capable of performing multiple loops as well. The results of the zero reflect experiment in Chapter 2 tell us that one second times should be possible. This corresponds to 5 loops or 10 mm$^2$ area. Only a factor of 3 would be needed in the $x$ oscillation amplitude to have the most sensitive gyroscope in the world.

A differential measurement will be most desirable to correct the phase instability. To do this some geometry is needed where fluctuations mentioned are common to two parallel interferometers and can be subtracted out while the rotation phase will be opposite in effect and thus add. This has been done in thermal non-guided wave atom interferometers to great success by using counter-propagating beams by the group of Kasevich.

This experiment has been a proof-of-principle experiment and in particular shows how useful it is to be able to scale a Sagnac interferometer from zero area. This feature allows us to diagnose potential problems as they become apparent, where in a fixed area initial system it could have been very hard to diagnose problems if the experiment were swamped with noise from the beginning. This technique could be used to produce areas on the order of the Kasevich experiment once these technical problems are eliminated. The ultimate
limits on the area enclosed are the coherence time of the atoms rather than the physical size of the device. We have demonstrated one second coherence times in the previous chapter so there is a lot of potential yet to be realized.
Chapter 5

DC Polarizability

In this application of the interferometer we seek to measure the effect of electric fields on $^{87}$Rb atoms. An external electric field $E$ will induce a dipole $p = \alpha E$ in a neutral atom, because the atoms electron distribution can be displaced relative to its nucleus. There is an energy associated with a fixed dipole in an electric field: $U = -Ep$. The energy associated with an induced dipole involves adding up the energies involved in its creation. An infinitesimal dipole $dp$ has the associated energy $dU = -Edp = -\alpha EdE$. Integration gives the total energy as

$$U_E = -\alpha \frac{E^2}{2}$$

(5.1)

This energy shift can create a phase for the interferometer to measure

$$\phi_E = -\frac{\alpha}{2\hbar} \int E^2(t)dt$$

(5.2)

assuming the $E^2(t)$ varies slowly in time compared to any electronic transitions.

There are many fundamental reasons to measure the polarizability $\alpha$ besides knowledge of just its value. Theorists use the polarizability to test ab initio calculations of atomic dipole matrix elements: $p_\psi = e \langle \psi | r | \psi \rangle$ [73, 74]. These dipole calculations are then used to determine van der Waals interactions [74], electronic state lifetimes [75], indices
of refraction [76], and blackbody shifts [77]. Polarizability experiments and theoretical calculations therefore remain an active field [78]. The blackbody shift is of particular importance because presently, it is responsible for the greatest uncertainty in atomic clocks [79] (see Chapter 1) and can cause unwanted transitions in Rydberg atoms [80]. The shift originates from the blackbody optical field of a finite temperature container around the atoms and this optical field will cause a stark shift of the energy levels. The blackbody shift manifests itself in all experiments since even without physical walls, there is the ever-present cosmic background mentioned in Chapter 1.

The best measurement for Rubidium, until very recently, was that of Molof et al. [81] using beam deflection more than 35 years ago and has a 2% error. A few months ago Holmgren et al. [82] demonstrated a direct measurement to about 1% using an atom-beam interferometer. They also could measure ratios of polarizabilities to about 0.3% by using the same apparatus with different species thereby eliminating some systematic errors. This enables a measurement of Rb to 0.4% when used with the Sodium measurement of Ekstrom et al. [83] giving $\alpha = 318.8/\pm 1.4 a_0^3$. We hope the BEC interferometer presented here could directly measure $\alpha$ to better than 0.1% accuracy.

<table>
<thead>
<tr>
<th>Value [$a_0^3$]</th>
<th>Source</th>
<th>Type</th>
</tr>
</thead>
<tbody>
<tr>
<td>319 ± 6</td>
<td>[81]</td>
<td>experiment</td>
</tr>
<tr>
<td>318.8 ± 2.8sys ± 0.8stat</td>
<td>[82]</td>
<td>experiment (absolute)</td>
</tr>
<tr>
<td>318.8 ± 1.4</td>
<td>[82]</td>
<td>experiment (ratio)</td>
</tr>
<tr>
<td>320</td>
<td>[84]</td>
<td>theory</td>
</tr>
<tr>
<td>309</td>
<td>[85]</td>
<td>theory</td>
</tr>
<tr>
<td>316.4</td>
<td>[74]</td>
<td>theory</td>
</tr>
<tr>
<td>318.6 ± 0.6</td>
<td>[74]</td>
<td>combination</td>
</tr>
</tbody>
</table>

In the table above, one can see values for the polarizability reported in the literature. The values are given atomic units of the Bohr radius $a_0$ but can be converted to SI units of Hz/(V/m²) by multiplying by $4\pi\varepsilon_0 a_0^3/h = 2.4883 \times 10^{-8}$. Note that there is
some disagreement between theory and experiment in the third decimal place. Theory is
combined with dipole matrix experimental data for the last value on the table.

The main difficulty in a measurement is accurate knowledge of the electric field. The
field is produced by applying some known voltage to a conductor geometry around the
atoms. It is the precise geometrical knowledge of the conductors that is key because
accurate measurements of voltage are relatively straight forward. This is similar to the
problem of measuring Newton’s constant, “big G”, where one must know precisely the
geometry of the test masses used. With respect to the geometry, a BEC interferometer
should have a significant advantage, because the atom wave packet is so highly localized.
This means that one can know accurately where it is when the electric field is applied. Also,
the BEC is so controllable that we can insert the BEC into a more simple geometry and
position it far away from regions with poorly understood electric fields at for example the
edge of conductors. In contrast a beam interferometer must pass through the conductors
and sample electric fields throughout the apparatus.

5.1 Theory

Calculating the polarizability is difficult and best left to theorists so this section will only
make a few general points.

For Hydrogen, the polarizability is relatively straight forward. We can use time-
independent perturbation theory. The first order perturbation energy of the ground state
$|nlm\rangle = |100\rangle$ (principle quantum number $n$, angular momentum $l$ and magnetic sublevel
$m$) is $U^{(1)}_{100} = \langle 100 | e E_z | 100 \rangle$ for a uniform electric field pointing in the $z$ direction. $|100\rangle$ is
spherically symmetric so the expectation value is zero. One can argue this directly from
the Wigner-Eckart theorem as well. The second order perturbation energy is:

$$U^{(2)}_{100} = \sum_{nlm \neq 100} \frac{e^2 E^2 |\langle nlm | z | 100 \rangle|^2}{U^{(0)}_{100} - U^{(0)}_{nlm}}$$

(5.3)
There is a sum over the dipole matrix elements, \( p = e \langle nlm | z | 100 \rangle \), of all possible states of the atoms. To relate this back to the polarizability, \( U^{(2)}_{100} = U_E = \alpha E^2 / 2 \) from Eq. (5.1).

We can see now how a model that predicts the matrix elements can be checked by the polarizability. For Hydrogen this is simple because we know the states \(|nlm\rangle\). We find \( \alpha_H = (18/4)a_0^3 \).

For multi-electron atoms this calculation is significantly more difficult. Relativistic effects have to be taken into account as well as the effect of the other electrons. Theorists often split the polarizability into pieces \( \alpha = \alpha_v + \alpha_c + \alpha_{cv} \) [73, 74]. Here, \( \alpha_v \) represents the contribution of the valence electron coupling to higher states and is similar to Eq. (5.3) [73], which for alkalis is dominated by the principle \( P_{1/2} \) and \( P_{3/2} \) states. These account for over 90% of the total polarizability. The second term represents the contribution from all of the other “core” electrons and contributes a few percent. The last term is coupling between the core and the valence states and is about 0.1%. Measurements that are accurate to the 0.1% level or better will be particularly useful for comparison.

This was all discussing the effect on a single atom. One could ask whether there will be any collective effects of a dense sample of atoms. There could be dielectric effects, following the treatment by Jackson [86]. The induced dipoles \( \mathbf{p} = \alpha \mathbf{E} \) make a density dependent polarization field \( \mathbf{P} = n \mathbf{p} \). This induces another electric field \( \mathbf{E}_i = \mathbf{P} / 3\varepsilon_0 \) (the near field that Jackson refers to should be zero here because the density distribution is symmetric). The average polarization will take the form

\[
\mathbf{P} = n\alpha(\mathbf{E}_0 + \mathbf{E}_i) \tag{5.4}
\]

substituting \( \mathbf{E}_i \) we can solve for \( P \). Remember that the dipole is the polarization per density so

\[
\mathbf{p} = \alpha \frac{1}{1 - n\alpha/3\varepsilon_0} \mathbf{E}_0 \tag{5.5}
\]
we can see that the polarizability of a free atom is modified to

\[ \alpha' = \alpha \frac{1}{1 - n\alpha/3\epsilon_0} \tag{5.6} \]

For the density in this experiment of around $10^{-9} \text{ mm}^{-3}$, the apparent polarizability increases by a fraction $2.5 \times 10^{-10}$: a negligible amount.

### 5.2 AC Polarizability

Before we delve too deeply in the static polarizability, it is worth mentioning the previous measurements we’ve done for the energy shift due to oscillating electric fields from a laser beam. Also known as the light shift, this ac polarizability can be particularly troublesome for some measurements. For example, optical clocks hold atomic samples in a lattice of optical traps formed from multiple standing waves. The light shifts the energy of the atoms and leads to a systematic error. This can be alleviated to some degree by using light that shifts the two states involved in a clock by the same amount. These so called “magic” wavelengths [87, 88] cancel the first order contribution to the clock, but there may be higher order contributions [88, 89]. Accurate knowledge of these shifts could prove useful for clocks or other experiments using optical traps.

This work was described in the thesis of B. Deissler [43] along with the article [90] so less detail will be presented here. Briefly, the energy shift expected from light of intensity $I$ is

\[ U_{ac} = -\frac{\alpha_{ac} I}{2\epsilon_0} \tag{5.7} \]

where $\alpha_{ac}$ is dependent of the frequency of the light $\omega_l$, the initial state of the atom and all a sum over all other states

\[ \alpha'_{ac} = \frac{2}{\hbar} \sum_{f \neq i} \frac{\omega_f}{\omega_f^2 - \omega_l^2} |p_{if}|^2 \tag{5.8} \]
When the clouds are at their maximum separation, a laser beam is directed at one atom wave-packet, shifting its energy and phase. A razor blade is imaged onto the atoms to keep the other wave-packet from being perturbed by the light.

where $\omega_i$ and $\omega_f$ are the energies in frequency units of the initial and possible final states.

The dipole matrix elements $p_{if} = \langle f|e z|i \rangle$ show up here too so the ac polarizability could also test atomic models. We can re-express the dipole matrix elements in terms of lifetimes, $\Gamma_{fi}$ and dipole moment ratios to find

$$\alpha_{ac}^{(i)} = \frac{4\pi\epsilon_0 \Gamma c^3}{2\omega_{33}^2} \sum_{f \neq i} \frac{\omega_f}{\omega_f^2 - \omega_i^2} \left| \frac{p_{if}}{p_{33}} \right|^2$$

where we’ve used the cycling transition between the $5P_{3/2}$ $F = 3, m_F = 3$ and $5S_{1/2}$ $F = 2, m_F = 2$ states with $\Gamma = 2\pi \times 6.065$ MHz. The dipole ratios can be calculated with angular momentum algebra [25, 91].

The experiment worked by illuminating one packet of the interferometer but not the other by imaging a razor blade onto the plane of the interferometer. The same camera used to take absorption images could be used to calibrate the intensity, and the camera was calibrated by a power meter. This is shown schematically in Figure 5.1.

We performed the experiment at three different wavelengths: 780.0 nm, 808 nm and on the $5P_{3/2}$ $F = 3, m_F = 3$ resonance at 780.24. The data are shown in Figure 5.2.

The interference signal decays with the intensity-time product from optical gradients.
5.2. AC POLARIZABILITY

Figure 5.2: AC Polarizability Data (taken from [43])

Shown here are interference data and fit for 780.0 nm (a), 808 nm (b), and through the 780.246 nm resonance(c). Atoms are lost due to resonant scattering depending on the detuning from the transition. The atom loss is shown directly in (d) and the loss of visibility in the interference signal is shown in the inset of (c).

The beams are Gaussian beams so there will be some spatial phase dependence similar to that discussed in Chapter 2. From the fits to the data we are able to determine 

\[ |\alpha_{780}| = 4\pi\varepsilon_0 \times (8.37 \pm 0.24) \times 10^{-25} \text{ m}^3 \]

and

\[ |\alpha_{808}| = 4\pi\varepsilon_0 \times (9.25 \pm 0.24) \times 10^{-25} \text{ m}^3 \],

where the errors are statistical from the fits. The calculation gives

\[ |\alpha_{780}| = 4\pi\varepsilon_0 \times (8.67) \times 10^{-25} \text{ m}^3 \]

and

\[ |\alpha_{808}| = 4\pi\varepsilon_0 \times (9.78) \times 10^{-25} \text{ m}^3 \].

The measurement is accurate to about 3% statistically but the calibration of the intensity yields an error of about 6%. This is the most precise method to date. The previous best was 8% by Kadar-Kallen et al. [92]. The observation of the light shift (dispersion) through resonance has been observed by using microwave spectroscopy of ground state hyperfine levels [93], but this technique can be used for atoms that lack hyperfine structure in the ground state, true of several species.
being considered for optical clocks.

5.3 DC Apparatus

For a static electric field one must know precisely the geometry of the conductors used in the same way as the intensity was needed in the AC case. To this end, the geometry in this experiment is kept simple by using a parallel plate capacitor configuration for which an analytic solution for the field exists. The geometry of the field plates is shown in Figure 5.4. We will also take advantage of the optical properties of the capacitor. Two parallel reflective surfaces make an etalon. Light of the correct wavelength shining through the plates (perpendicular to the conducting plane) will be reflected many times in the optical cavity between the conductors. Light intensity increases in the cavity and some of it will pass through to the other side. Only light that has a wavelength that is an integer multiple of the twice the separation distance between the plates will be transmitted through the etalon. We can measure wavelength to 5.5 digits with a Coherent Wave Master wavelength meter. Thus a precision for the field at about this level may be possible. The conductors will be made from gold because its reflective at 780 nm, a good conductor, and vacuum compatible.

The conductors are made from a 25 nm thick layer of gold evaporatively deposited onto 1 mm thick glass plates with a 5 nm thick layer of Titanium used a bonding agent between the gold and glass. One of the plates is chemically etched with a 150 µm deep channel before the gold deposition to create the plate separation needed. The conductors are in the shape of a “T” and one of the conductors is asymmetric so that the ends of the T can stick out from the opposite plate. In the 1 mm square area that is clear of the opposite plate, a 5 µm thick layer of gold is deposited to receive electrical connections to a voltage source. We make these connections via ultrasonic wire bonding of a gold wire to another larger gold pad nearby. Wire bonding was done by Theodore Reck in the University of Virginia
Figure 5.3: Polarizability Experiment Schematic
The schematic on top demonstrates how the field will be applied. The BEC will begin at the symmetry point between two conductors above and below, one of which is split into two isolated conductors. The Bragg beam is applied parallel to the conductors (left and right in the picture) in such a way as to move the wave-packets away from the gap and into independent electric field regions. An electric field is applied at the maximum separation of the atoms in the interferometer sequence. The lower figure shows the timing of the experiment. During a two-reflect interferometer, the electric field is applied twice to the same wave packet by switching the applied voltage between the two field plates.
Figure 5.4: Capacitor Schematic
(color) The plate with the two conductors will be made slightly asymmetric as shown and placed on top of another plate that has been etched 150 µm deep. When folded on top of each other, the plates are positioned askew to allow room for wire bonds to each of the conductors. There will also be some optical access along the front face.

Engineering School. Scaled drawings of the conducting surfaces are shown in Figure 5.5. In the figure, one can see that the upper plate has a 40 µm gap between two isolated conductors. One packet of atoms will be positioned above each of these conductors when the electric field is applied. This is the asymmetric conductor and sits on the bottom of the apparatus relative to gravity. The other conductor is a single plane and this plate is the one for which the glass was etched 150 µm deep before deposition. The capacitor is fastened to a mount in the vacuum chamber made from ceramic Macor from 3M Company shown in Fig. 5.6. The capacitor is bolted to the mount with a Vespel clamp and two beryllium copper spring washers held down with #0-80 screws. The capacitor is at a 45 degree angle to the mount to make it in line with the external waveguide. The mount also holds the secondary gold pads at the other side of the wire bonds. These pads are made from gold electroplated onto copper sheet and they are bolted to the ceramic with #00-90 screws: 2 pads on the top and 1 on the bottom. The ceramic mount attaches to
an Aluminum arm that bolts in turn to the vacuum flange that also bolts to the vacuum chamber. The complete assembly is shown close up in Fig. 5.7.

5.4 Capacitor Construction Procedure

The actual procedure for creating the capacitor plates is a done in a clean room. This work was done in collaboration with the Engineering School at the University of Virginia under the direction of Nathan Swami with students Adrea Robinson and Robert Hunter. The following is an example methodology for creating samples in their clean room, in this case for the flat field plate with two electrodes. The first step is to clean the glass plates to be used. The glass is washed liberally with ethanol, d-limonene, and methanol in that order. Note, acetone cannot be used because it is not allowed in the deposition room. Finally, the glass is air dried on a hot plate at 120°C.

The glass is then primed with Hexamethyldisilazane (HMDS). The primer is applied in a vapor chamber for even coating. The primer is useful for attracting the photo resist in the next step.

The photo resist is applied on a spinner. A liberal amount of resist AZ® 5214 is applied to the primed glass plate and any bubbles removed. The spinner runs at 5000 rpm for 30 seconds. Here a potential problem is that the glass can become dislodged and crash into the spinner walls. Smaller glass plates would be better in this regard, but then the sample production rate will decrease.

Assuming the samples survive the resist application, the next step is to apply ultraviolet light to the resist through a mask. The mask blocks out light in the same pattern of the eventual gold. The mask is black ink on a transparency printed in high 4000 dpi resolution. It was plotted in Autodesk Inventor, a CAD program, and printed by Pixels, a professional printing business. The mask is placed ink side down and exposed for 18 seconds.
Figure 5.5: Field Plate Drawings
These are scaled drawings of the gold conducting surfaces. The samples are made in batches of 16.
The next step is to develop the exposed resist. AZ® 400k developer is diluted 1:4 with de-ionized water. The glass is immersed in the developer mixture. At this point one can see the negative of the mask emerge as the unexposed resist is removed.

The last step is to evaporatively deposit titanium and gold onto the sample. The evaporator operates at a pressure of $6 \times 10^{-6}$ torr. The first layer is Titanium, which helps the gold bond to the glass. The evaporator is set to deposit 5nm of Ti at a rate of 0.1 nm/s. The rate is set with a power setting (25% for this rate). Next at 38% power, which is 0.3 nm/s, 25 nm should be deposited in 8.3 seconds. The evaporator typically overshoots by a small amount. In this example, the evaporator overshoots and actually
deposits 5.9 nm of Ti and 25.4 nm of Au.

5.5 Plate Spacing Measurement

The etalon characteristic of the capacitor is used to measure the conductor spacing. Light for which twice the separation between the plates is an integer multiple of the wavelength will be transmitted. Figure 5.8 shows the transmitted profile of sending a large area beam at 780.24 nm (the Rb resonance) through the etalon towards a camera. In the figure, one can see two stripes of light at a diagonal to the picture that are the etalon fringes. The vertical, narrower strip of light is the isolating gap between the two independent conductors on the, flat, non-etched plate.

The plates are not perfectly parallel and in fact there is some warpage, probably as a result of the clamp holding them securely together. If the laser light is shifted in
wavelength, the relative position of the fringes moves to a position where the plate spacing is correct to transmit the new wavelength of light. It turns out that the two stripes in the figure are for the same fringe. That is to say the plate separation is equal in the two locations of the fringes. Shifting the wavelength can move the fringes toward each other where they eventually combine and two new fringes appear at the left and right edges in the picture.

To make an accurate measurement of the plate spacing in the vicinity of the interferometer (under the gap in the center of the picture), one will need to move the fringe to that location. One wavelength of light is not enough to determine the plate spacing because it is impossible to determine what integer multiple of wavelengths is present between the plates. To measure the plate spacing, the speed of light can be used as a meter stick. The condition for transmission is $n\lambda = 2t$ for wavelength $\lambda$, integer $n$, and plate spacing $t$. Dividing by the speed of light $c$, one can write this condition in terms of frequency $\nu$ as

$$\nu = \frac{n}{2t} c$$  \hfill (5.10)

Then if the wavelength (and frequency) is shifted so that light transmits again, then the integer changes by one. The frequency difference between these two fringes defines the free spectral range,

$$\Delta \nu = \nu_{FSR} = \frac{c}{2t}$$  \hfill (5.11)

If we shift the frequency over many fringes then we can accurately measure the free spectral range and the plate spacing.

Figure 5.9 shows the frequency measurements of many fringes. The slope gives the free spectral range and thus the distance between the plates. For this measurement, three lasers were used to give as broad a frequency range as possible. The Coherent MBR 110 Ti:Sapphire was tuned over its entire range to acquire the majority of data. The data was taken such that the reflection from the conductors was retro-aligned to better than
Figure 5.8: Etalon Fringe
This is the etalon fringe at 780.24 nm. The fringe appears in two places because the conductors are not perfectly parallel.
1 mrad. There are some frequencies where no data could be taken because atmospheric absorption lines of H$_2$O and O$_2$ blocked the laser light from reaching the apparatus. The error bars shown come from the uncertainty in positioning the fringe in the same place. This uncertainty relates to the finesse of the cavity. The finesse $\mathcal{F}$ is function of the power reflectivity $R$ of the mirrors used:

$$\mathcal{F} = \frac{\pi R^{1/4}}{1 - R^{1/2}}$$  \hspace{1cm} (5.12)

The reflectivity was measured to be about 75% for a finesse of about 20. Note the transmission is not $1 - R$ but instead about 11% for one plate because the Ti absorbs some of the light as well. Because the mirrors are not perfect, then the condition for transmission is not perfect. Light near a fringe of the cavity will also transmit and the full-width-half-maximum frequency spread is

$$\delta \nu_{\text{FWHM}} = \frac{\nu_{\text{FSR}}}{\mathcal{F}}.$$  \hspace{1cm} (5.13)

This is why the fringe in the picture is not infinitely narrow. The center of the fringe is determined as well as possible up to the resolution of the camera.

Two more data points were added using a HeNe laser at 633.0 nm and a frequency doubled diode pumped solid state laser at 532.4 nm. These lasers were not tunable so we cannot control the position at which they transmitted. The fringe position-optical frequency relationship was calibrated by observing how the fringes moved when the Ti:Sapphire was tuned. The HeNe was estimated to be at about 1/3 of a free spectral range and the Nd:YAG was nearer at 1/5 of a FSR. The nearest integer fringe number can be determined by finding the integer that puts the measurement nearest the line that fits the Ti:Sapphire data. The complete set can then be fit again to find the new slope. Both fits are shown in Figure 5.9. The Ti:Sapphire data fits to a free spectral range of 990.8 ± 0.3 GHz and the combined data gives 991.9 ± 0.2 GHz. This gives plate spacing measurements
Figure 5.9: Plate Spacing Measurement

Here is data for the frequency of many fringes. The slope is the free spectral range. On the left is data for all the frequencies available with a linear fit (dashed). On the right is the data from just tuning a Ti:Sapphire laser in the near infra-red where the Rubidium transitions lie with a linear fit (dotted). The residuals show the deviation from the fits to the above data and the opposite data’s fit for comparison.

of 151.28 ± 0.03 µm and 151.12 ± 0.02 µm respectively. These measurements do not agree within their measurement error and this is because the path length 2t depends slightly on wavelength. The dependence comes from the fact that the optical field penetrates some distance into the gold before being reflected. Related to the skin depth, this phenomenon was studied by Jiraphat Tiamsuphat, an undergraduate working in this lab. His calculation indicated that there could be up a 100 nm difference between the measured plate spacing from the infrared data acquired from the Ti:Sapphire and the green light of the solid state laser. This difference is similar to the 150 nm discrepancy between the two measurements. The addition of the extra laser wavelengths helped the precision only modestly because the measurement error from these extra lasers was relatively large compared
to the Ti:Sapphire and the extra lasers also introduced potential systematic effects. We, therefore, take the spacing measurement acquired from the Ti:Sapphire laser data alone.

5.6 Analytic Field Calculation

An analytic form for the electric field can be derived for two conductors at positions $z = 0$ and $z = t$ that have infinite extent in the $x - y$ plane. The conductor at $z = 0$ will be held at zero potential and the other conductor will have the boundary condition of being zero for $y < 0$ and a constant, $V_0$, for $y \geq 0$. Said another way:

$$\Phi(y, t) = V_0 \Theta(y) \quad (5.14)$$

where $\Theta$ is the Heaviside function. Here we are making two approximations. One that conductors are large in the $x$ and $y$ direction compared to any relevant length scale in the apparatus and that the gap between the split conductors has negligible width and that the electric field has no $x$ dependence. The approximation will be valid for wave-packet separations that are large compared to the actual gap size, but small compared to the size of the conductor plate. We will treat the case of a finite sized gap between the split later in the next section. Laplace’s equation $\nabla^2 \Phi = 0$ for the electric scalar potential, $\Phi(y, z) = Y(y)Z(z)$ is separable in this boundary value problem forming an equation, after dividing by $YZ$ of

$$\frac{Y''(y)}{Y} + \frac{Z''(z)}{Z} = 0 \quad (5.15)$$

The two pieces are independent so to be generally equal to zero we need

$$\frac{Y''(y)}{Y} = -k^2 \quad \text{and} \quad \frac{Z''(z)}{Z} = k^2 \quad (5.16)$$

for some constant $k$. The unbounded $Y$ direction differential equation has plane wave solutions. The other equation is a combination of hyperbolic sines and cosines except
the boundary condition \( Z(z = 0) = 0 \) forbids the hyperbolic cosine. Together we have a solution

\[
Y(y) = Ae^{iky} \quad (5.17)
\]

\[
Z(z) = B \sinh kz \quad (5.18)
\]

for some constants \( A \), and \( B \). Generally any linear combination of constants \( k \) solves the problem so for the full solution we must integrate over \( k \).

\[
\Phi(y, z) = \int_{-\infty}^{\infty} A(k)e^{iky} \sinh (kz)dk \quad (5.19)
\]

where the constant \( B \) was absorbed into the function \( A(k) \). We now recognize a Fourier transform for \( \sqrt{2\pi}A(k) \sinh (kz) \). We can insert the other boundary condition in Eq. (5.14), but we’ll do it on the other side of the Fourier transform i.e.

\[
\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \Phi(y, t)e^{-iky}dy = \sqrt{2\pi}A(k) \sinh (kt) = \frac{V_0}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \Theta(y)e^{-iky}dy \quad (5.20)
\]

The solution for the Heaviside function Fourier transform is

\[
\sqrt{\frac{\pi}{2}}\delta(k) - \frac{1}{ik\sqrt{2\pi}} \quad (5.21)
\]

where \( \delta \) is the Dirac delta function. Now that we have a \( A(k) \), we can form a complete integral solution of the potential as

\[
\Phi(y, z) = \frac{V_0}{2\pi} \int_{-\infty}^{\infty} \frac{\sinh (kz)}{\sinh (kt)} \left[ \pi\delta(k) - \frac{1}{ik} \right] e^{iky}dk
\]

\[
= \frac{V_0}{2} \left[ \frac{z}{t} - \frac{1}{i\pi} \int_{-\infty}^{\infty} \frac{e^{iky} \sinh (kz)}{k \sinh (kt)} dk \right] \quad (5.22)
\]

This integral is solvable by using the contour \( \Gamma \) in Fig. 5.10 over the \( \kappa \) complex plane.
Figure 5.10: Electric Potential Integral Contour
The contour followed depends on the sign of \( y \). The contour is in the limit of \( R \to \infty \) and \( \epsilon \to 0 \) to go around the pole in the integrand at \( \kappa = 0 \).

in conjunction with the residue theorem:

\[ \oint_{\Gamma} F(\kappa) d\kappa = 2\pi i \sum \text{Res} [\kappa_0; F] \]  

(5.23)

for poles at \( \kappa_0 \) in integrand function \( F \). With the contour in Fig. 5.10 the integral becomes for \( y > 0 \).

\[
\begin{align*}
\oint_{\Gamma} e^{iyy} \sinh (\kappa z) \frac{d\kappa}{\kappa \sinh (\kappa t)} &= I + \lim_{R \to \infty} \int_0^\pi e^{iyR \exp [i\theta]} \sinh (R \exp [i\theta]z) \frac{R \exp [i\theta]}{\sinh (R \exp [i\theta]t)} Ri \exp [i\theta] d\theta \\
&+ \lim_{\epsilon \to 0} \int_0^\pi e^{iy\epsilon \exp [i\theta]} \sinh (\epsilon \exp [i\theta]z) \\
&\quad \epsilon \exp [i\theta] d\theta \\
&= \int_{2\pi}^{2\pi} e^{iy\epsilon \exp [i\theta]} \sinh (\epsilon \exp [i\theta]t) \epsilon \exp [i\theta] d\theta \quad (5.24)
\end{align*}
\]

where \( I \) is exactly the integral in Eq. (5.22). For \( y < 0 \) the limits of the last two integrals goes to \( \int_{2\pi}^{2\pi} \) and \( \int_{2\pi}^{2\pi} \) respectively. The integrals over \( \theta \) work out to be \( \pm iz\pi / t \) for \( y \geq 0 \).

The last part that needs to be worked out is the sum over the residues that equal
Eq. (5.24). There will be residues for the poles at \( \kappa = in\pi/t \), where \( n \) is a positive integer for positive \( y \) and negative integer for negative \( y \). We have already effectively taken care of the pole at \( \kappa = 0 \) with the choice of contour. The residues are

\[
\text{Res} \left[ \kappa = \frac{in\pi}{t}, \frac{e^{iny} \sinh (z\kappa)}{\kappa \sinh (t\kappa)} \right] = \frac{t(-1)^n}{n\pi} \sin (n\pi z/t) e^{-n\pi|y|/t} \tag{5.25}
\]

Inserting the residues into the sum in (5.23) we can get an answer for the integral \( I \) in (5.24).

\[
I = (2\Theta(y) - 1) \left[ \frac{t}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^n}{n} e^{-|y|n\pi/t} \sin (n\pi z/t) - \frac{i\pi}{t} \right] \tag{5.26}
\]

inserting (5.26) into (5.22) we find the potential in the form of an infinite sum:

\[
\Phi = \frac{V_0}{t} \Theta(y) - \frac{V_0}{\pi} [2\Theta(y) - 1] \sum_{n=1}^{\infty} \frac{(-1)^n}{n} e^{-|y|n\pi/t} \sin (n\pi z/t) \tag{5.27}
\]

The sum converges as

\[
\sum_{n=1}^{\infty} \frac{(-1)^n e^{nx}}{n} = -\ln (1 + e^x) \tag{5.28}
\]

giving the analytic form

\[
\Phi = \frac{V_0}{t} \Theta(y) + \frac{V_0}{2\pi i} [2\Theta(y) - 1] \ln \left[ \frac{1 + \exp (-\pi|y|/t - i\pi z/t)}{1 + \exp (-\pi|y|/t + i\pi z/t)} \right] \tag{5.29}
\]

We can simplify using the relations \((\kappa - \kappa^*)/2i = \text{Im}(\kappa)\), \( \ln [x + iy] = \ln |x + iy| + i\text{arg}(x + iy) \) and \( \text{arg}(x + iy) = \arctan(y/x) \) to find

\[
\Phi = E_0 z \Theta(y) - \frac{E_0 t}{\pi} [2\Theta(y) - 1] \arctan \left[ \frac{\sin (\pi z/t)}{1 + \exp (-\pi|y|/t) \cos (\pi z/t)} \right] \tag{5.30}
\]

where \( E_0 = V_0/t \) is the field in an infinite parallel plate capacitor. The electric field is the gradient of the potential and the atom energy (and phase) scale with \( |\nabla\Phi|^2 \) so in this ideal case we have a solution for the phase of the interferometer. One can see that for an accurate measurement we need values for the voltage applied \( V_0 \) as well as the already
measured value of $t$. We will also need values for the position $y$ and $z$ if the atoms are near the gap in the split conductor. For positions $y \gg 0$, the field becomes the uniform $E = E_0$.

The magnitude of the square field is shown in Figure 5.11 along with the gradients in $E^2$. The gradients in $E^2$ are the main source of problems in this experiment for two reasons. First, they are proportional to the force and will pull the atoms towards the split conductor gap where the field is the highest. Second, they introduce phase gradients that will reduce the visibility of the interference signal in a similar way as was described in Chapter 2.

In addition to the analytic calculation we did a numerical calculation which took into account the finite sized gap in the split conductor of 40 $\mu$m. This calculation was done with a program called Ansys using a finite element analysis and the program code can be found in the Appendix. The numerical gradient data was taken by simply subtracting one $y$ position data point from its neighbor one micrometer away. The noise reveals some of the round off error in the numerical method. One can see that the inclusion of the gap lowers the gradient near $y = 0$ under the gap, while increasing it further away. There is a point between 30 and 40 $\mu$m where the transition occurs. The larger the gap the further this point moves away. Having a large gap allows for larger voltages to be applied without leakage current between the conductors but has a negative effect on the interferometer at larger separations. The gradient in the $z$ direction of the field is asymmetric about $y = 0$. This is problematic because the difference will cause a net gradient in the phase difference when the atoms are recombined and this will decrease the visibility of the experiment. The gradient in the $y$ direction becomes asymmetric about $y = 0$ when the atoms move away from $z = 0$ in the middle of the field plates (note $z = 0$ is referenced to the lower field plate in the analytic calculation). For the best performance the interferometer should begin as close as possible to the center.

A 40 ms interferometer will sample the field at 117 $\mu$m where the uniformity is at the part per thousand level. The best interferometer we can hope for is an 80 ms interferometer at $y = 234$ $\mu$m and part per ten thousand uniformity. The plate spacing measurement is
5.6. ANALYTIC FIELD CALCULATION

Figure 5.11: Electric Field in Capacitor

The upper plot shows the analytic and numerical values of the electric field for three vertical $z$ positions along the $y$ axis. The lower plots shows the magnitude of the gradient of the square electric field along the $y$ (left) direction and $z$ direction (right, analytic only) as a function of the $y$ direction for the same three vertical positions. The kink in the $y$ gradient at $z = +10 \, \mu m$ data is where the gradient goes through zero and becomes negative. Note the gradient is no longer symmetric about $y = 0$ when the atoms are away from the vertical center. The $z$ gradient is asymmetric about $y = 0$ everywhere.
good to about that same precision. It is hoped between these two pieces of information, a measurement of the polarizability could be made to three or four decimal places, which would be the most accurate direct measurement ever made.

5.7 Atom Loading

Laser cooled atoms from the MOT are brought near the capacitor apparatus via the translating magnetic trap described in the introduction. We’d like to make the BEC as near as possible to the capacitor to make moving the BEC into the electric field region as easy as possible. The atomic cloud is too large in the magnetic trap so the first stage of evaporative cooling is done about a centimeter away from the capacitor. The now smaller and colder atomic cloud is moved closer to the capacitor by moving the magnetic trap coils with a second motorized linear stage on an axis about 45 degrees to the larger translation stage previously described. This is schematically shown in Figure 5.12.

The BEC is made about 1 mm away from the capacitor and must be inserted between the field plates. To control this maneuver an additional pair of magnetic field coils is attached coaxial to the waveguide. They add a static $y$-bias field that can move the zero of the static spherical quadrupole along the $y$-direction. The $y$-bias coils are 30 turns each of 21 gauge wire located 70 mm from the capacitor. They are wrapped directly onto the guide making a rounded square shape about 75 mm on the $x$ side and 65 mm on the $z$ side. The two make a total magnetic field of 1.1 Gauss/Amp. We load the capacitor with the spherical quadrupole at 13 gauss/cm so the $y$-bias field will move the atoms at 0.9 mm/Amp. In this trap, the atoms are extend less than 10 $\mu$m along $z$, compared to the 151 $\mu$m space between the conductors.

Loading the atoms is complicated by eddy currents in the field plates and the surrounding conductors, particularly the gold-plated copper pads that connect the voltage sources to the field plates. The eddy currents originate from the ac magnetic fields comprising
5.7. ATOM LOADING

Figure 5.12: Positioning the BEC near the Capacitor

The magnetic trap moves laser cooled atoms to the left of the capacitor as shown in this birds eye view of the experiment. A second stage moves the atoms diagonally to a position in front of the capacitor and on the wave guide axis while the first stage of evaporation is happening.

The rotating bias field and linear quadrupole. The effect of these eddy currents is that the magnetic field is slightly lower inside the field plates (the field is shielded slightly) and higher outside. Without the $y$-bias field, as the spherical quadrupole is reduced the atoms are pushed away from the field plates. The actual loading process is a complicated series of ramps of all four magnetic field sources: the $y$-bias, the linear and spherical quadrupoles, and the rotating bias as shown in Table 5.2. After the atoms are loaded near the gap in the split conductor the spherical quadrupole is reduced to 6 gauss/cm and the $y$-bias field is reduced to keep the $y$ position constant. The final trap used had trap frequencies
Figure 5.13: Atom Loading
(Top) The atoms are loaded into the capacitor along the multistep path shown with the magnetic field ramps discussed in the text. The atomic absorption is indicated with a box, but also notice there is a reflection of the absorption in the top plate. Gravity pulls up in the figure. (Bottom) This is an image of the atoms positioned under the gap. The gap is highlighted with parallel lines.
5.7. ATOM LOADING

Table 5.2:
Loading Sequence using the spherical quadrupole: \( dB_{SQ}/dy \), \( y \)-bias: \( B_y \), linear quadrupole: \( dB_{LQ}/dz \), and the rotating bias: \( B_0 \). Blank spaces indicate no change.

<table>
<thead>
<tr>
<th>ramp time [s]</th>
<th>( dB_{SQ}/dy ) [gauss/cm]</th>
<th>( B_y ) [gauss]</th>
<th>( dB_{LQ}/dz ) [gauss/cm]</th>
<th>( B_0 ) [gauss]</th>
</tr>
</thead>
<tbody>
<tr>
<td>TOP</td>
<td>180</td>
<td>0</td>
<td>0</td>
<td>4.2</td>
</tr>
<tr>
<td>1</td>
<td>45</td>
<td>31</td>
<td>4.2</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>23</td>
<td></td>
<td></td>
<td>20.4</td>
</tr>
<tr>
<td>4</td>
<td>13</td>
<td>1.6</td>
<td>2.7</td>
<td></td>
</tr>
<tr>
<td>3.5</td>
<td></td>
<td>2.9</td>
<td>18.3</td>
<td></td>
</tr>
<tr>
<td>0.8</td>
<td></td>
<td>4.1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>11</td>
<td>3.1</td>
<td>30.4</td>
<td></td>
</tr>
<tr>
<td>2.5</td>
<td>8</td>
<td>2.8</td>
<td>30.2</td>
<td></td>
</tr>
<tr>
<td>2.5</td>
<td>6</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

of \( \omega_x = 2\pi \times 6.3 \text{ Hz}, \ \omega_y = 2\pi \times 2.0 \text{ Hz}, \text{ and } \omega_x = 2\pi \times 6.0 \text{ Hz} \). The trap is a tighter than previous experiments to keep the atoms from spreading out too much. The cloud size has Thomas-Fermi radii of approximately \( L_x = L_z = 10 \mu m \) and \( L_y = 30 \mu m \). This gives more room for the atoms in the capacitor and makes them easier to manipulate with the standing wave.

The loading sequence is shown graphically in at the top of Figure 5.13 up to the point where the atoms can be seen from the side. Here the probe beam shines along the \( x \) direction and the glass substrates for the gold conductors are shown in profile. The split conductor (top plate in the figure) has the gap just past the viewable area (on the right as shown). The inability to observe the atoms’ vertical position when they are under the conductor gap turned out to be a serious weakness of the design as will be discussed. The atoms are taken carefully around the corner of the plate because it was observed that the atoms would be pulled towards the corner and lost. It has not been determined what this effect is, but it appears highly localized to the corner.

The bottom picture in Figure 5.13 shows the view from above through the capacitor. This view is difficult to use because the light and atomic absorption shadow must pass
through the conductor and glass substrates, but one can use this view to verify the $y$
position of the atoms relative to the conductor gap. It also can verify that the etalon
fringe has not changed thus providing an *in situ* check of the plate spacing $t$. This is
important because other experiments measure the geometry before and after vacuum and
assume it doesn’t change once in vacuum.

One more note on loading the atoms, the loading sequence in sensitive to changes in
the local magnetic fields of the system as well as changes in the tilt of the optical table.
For this reason, the passive vibration damping system for the optical table was not used
in this experiment. An active vibration cancellation stage (described in Chapter 3) was
used to isolate just the standing wave retro-reflection mirror.

## 5.8 Bragg Beam in Capacitor

With the atoms positioned between the field plates, the next step is to test the standing
wave operations. The beam needs to be uniform over the BEC for the standing wave
operations to work properly. The beam must be compressed to a focus that is small
enough to clear the aperture of the field plates to avoid diffraction effects, but not so
small that the intensity varies across the BEC because of the Gaussian profile of the
beam. This turns out to be a tremendous challenge.

Several optical systems were tested. One system used a spherical lens to produce a
50 $\mu$m waist spot. This produced very poor results, because the standing wave operations
did not work. A second attempt used no focusing elements: a collimated beam was aimed
at the BEC instead. This worked better but still produced poor results. Figure 5.14 shows
the final system that produced the best results. A cylindrical lens produced a light sheet
with a 60 $\mu$m focus in the $z$ direction and a 1 mm size in the $x$ direction. The exiting
beam was collimated by a spherical lens, and then focused onto a mirror with second lens.
The combination of these two lenses served to image any diffraction pattern that emerged
Figure 5.14: Bragg Beam Optics for Field Plates
A schematic of the Bragg beam optical setup. The cylindrical lens makes a 60 \( \mu \)m waist sheet of light that travels through the capacitor. This light is re-collimated by a spherical lens and then focused onto a mirror. The two spherical lenses also make telescope that images the capacitor plane back onto itself.

from the capacitor back onto the capacitor. It also allowed the mirror to be positioned far enough way to fit on the active vibration isolation stage.

The standing wave operations worked in this configuration but only just. Part of the problem seemed to be inconsistent positioning of the atoms, especially vertically. To aid the loading sequence a far off resonant optical trap or FORT was attempted. A FORT uses a detuned travelling wave to create a trapping potential via the AC stark shift. The atoms will be attracted to the focus of the FORT. This uses similar detuning to the standing wave so the same laser could be used for both. The idea is to have the standing wave perpendicularly polarized to the FORT beam and use a polarizer to block the FORT beam before the retro-mirror. If successful the atoms would be trapped in the same position as the standing wave so the consistency of the standing wave operations would improve. What we found was evidence as to how poor the beam quality of the standing wave was.

As the FORT strength is increased the atoms should move to the new global potential
Figure 5.15: FORT Trap Diffraction
On the left we see atoms being pulled into the FORT trap from their initial position in the magnetic trap as the FORT trap strength is increased. At 0.3 mW of power, some phenomenon disrupts the loading. The BEC breaks up into pieces. The plot on the right shows as a function of time held in 0.3 mW FORT trap, the positions of the BEC wave-packets. The linear fits indicate velocities of $3.4 \pm 0.2$, $0.29 \pm 0.02$, $-0.008 \pm 0.004$, $-0.21 \pm 0.026$, and $-1.4 \pm 0.2$ mm/s.

minimum until the FORT strength dominates the magnetic trap strength. In Figure 5.15 one can see this begin to happen, but then at a critical power of 0.3 mW, the BEC breaks up into many pieces moving at different velocities in the vertical direction. They appear to be diffracting from some intensity pattern in a similar way to the standing wave operations. This was done in front of the capacitor with respect to the direction of the light. Evidently light reflects from the capacitor edge and creates a complicated intensity pattern. The FORT trap was abandoned, but it has identified the main experimental problem.

5.9 Results

The interferometer could only be made to work for very short times. At longer times, the standing wave operations, in particular the second reflect and recombination pulses caused a large loss of atom number. It’s possible that the atoms were diffracting as we saw in the FORT case and crashing into the field plates. Figure 5.16 shows the interference signal
5.9. RESULTS

Figure 5.16: Interferometer Inside Capacitor
This is the interference observed when using the Bragg beam frequency to shift the phase for 8 ms of total time.

after 8 ms in a double reflect interferometer when the Bragg beam frequency is shifted as described earlier.

The visibility is only about 40% in the 8 ms interferometer. The wave packets only move about 23 µm from the starting position, so the electric field will not be uniform. The electric field was applied by sending a sine wave with a period of $T_E = 100 \mu s$. The phase one would expect for this is

$$\phi_E = -\frac{\alpha}{2\hbar} \frac{E_0^2 T_E}{2} f^2(y, z)$$

(5.31)

where $f$ contains all of the spatial variation. The gradients in $f$ will tend to push the atoms around. To combat this, the field is applied twice, once just after the first reflect and once just before the second reflect. From the analytic solution we have, one can predict what the actual phase difference will be.

$$\Delta \phi_E \approx -\frac{\alpha T_E E_0^2}{\hbar} \left( \frac{\pi y}{l} \right) \left[ 1 - \frac{\pi^2 z}{l^3} - \frac{\pi^3 z^2}{l^5} \right]$$

(5.32)

valid for $y <<< t$, where this interferometer is unfortunately operating. One can determine
the visibility as well in the same way as was done in Chapter 2. The visibility $V$ is

$$V = \int n(\mathbf{x}) \cos[\phi_E(\mathbf{x})] d^3x$$ \hspace{1cm} (5.33)$$

for

$$n = \frac{N}{L_x L_y L_z} \left[ 1 - \frac{x^2}{L_x^2} - \frac{y^2}{L_y^2} - \frac{z^2}{L_z^2} \right]$$ \hspace{1cm} (5.34)$$

where $N$ is the number of atoms, and $L_i$ are the Thomas-Fermi lengths.

Figure 5.17 shows the data for several attempts. The data is plotted as a function of the peak-peak applied voltage squared, which is proportional to the phase. The data in (a) is for two runs where the order of conductors charged is reversed. One expects the phase to switch by $\pi$ in this case. Here they are plotted on the same axis with a negative sign since the phase difference will switch sign. Note the offset phase for the negative data matches the test case in Fig. 5.16. The average $\langle N_0/N \rangle$ is also similar, but the peak visibility is reduced to about 10% then decays with larger applied voltages due to phase gradients.

$$\frac{N_0}{N} = \left< \frac{N_0}{N} \right> + AV(y, z) \cos(\phi_0 + \Delta \phi_E(y, z))$$ \hspace{1cm} (5.35)$$

where $\langle N_0/N \rangle$, $\phi_0$, and a multiplier $A$ for the visibility is given from the exponentially modified cosine wave fit.

The values for the polarizability $\alpha$ for these data can be determined from the period of oscillation as measured by a fit to a sine wave. Unfortunately, this gives a value an order of magnitude higher than the expected value of $319 a_0^3$. With the separation of only 23 $\mu$m the atoms are still on the same distance scale as the gap in the split conductor and in a region where the electric field has a strong spatial dependence. To accurately know the electric field requires knowledge of the position and we can’t determine the vertical position.

Turning this around, we can use the expected value for $\alpha$ to determine where the
5.9. RESULTS

Figure 5.17: Polarizability Interference
Plotted here are interference curves made by applying electric fields. (a) and (b) are from two different days. In (a), the negative values were made by applying the electric field to opposite wave-packet as was used for the positive. The crossed out data points are suggested to be thrown out.
Figure 5.18: Predicted Polarizability Interference
The data is the same as the positive side of Fig. 5.17(a). The dashed curve is a fit to that data of a sine wave multiplied by an exponential decay. The solid curve is the predicted interference signal for a BEC initial position of $y = 0$ and $z = 60 \, \mu m$ assuming the offset in $N_0/N$ and $\phi_0$ along with the underlying visibility predicted by the exponential fit.

atoms are. Figure 5.18 shows the first data set with a fit to a cosine wave modified by an exponential decay. With this we can numerically solve Eq. (5.33) for the visibility and predict for various positions the interference signal:

Good agreement is found for $z = 60 \, \mu m$. This is very near the split conductor plate. In the past we’ve observed the pulses work better near one of the plates, so it’s entirely plausible this is what has happened. Figure 5.19 shows the visibility function at this position.
Figure 5.19: Predicted Visibility for Electric Field

The electric field gradients will cause phase gradients and a similar loss of visibility as was seen in Chapter 2. This curve was taken from the analytic field for a BEC starting position at $y = 0$ and $z = 60 \, \mu m$.

5.10 Outlook

These results are not nearly as good as hoped for obviously. For an accurate measurement the atoms need to be separated farther away from the conductor gap where the field becomes uniform. In that case, only modest knowledge of the atoms position is required for accurate results. It seems that the atom wave packet is small and localized enough to fit into the capacitor, but the Bragg standing wave is not. If no way of compressing the beam can be found, then the capacitor plates will have to be rebuilt with greater plate spacing. The electric field becomes uniform when $y \gg t$ so if the separation $t$ is made larger than the atom separation must also be larger. This will require longer interferometer times than are available presently. A differential measurement of some type will likely be required to attain these times in order to cancel out vibration noise. Another solution might be an extremely well isolated platform. In any event, we have learned one important lesson:
Despite the good localization of a Bose-Einstein condensate, if one wished to use standing wave manipulations, then one must consider the significantly larger space needed for its operation.
Chapter 6

Conclusion

6.1 Summary

In this work a Bose Einstein condensate guided matter-wave interferometer has been presented that shows much promise as a useful scientific instrument. Also shown is how the same mechanism that gives the device much of its utility, the wave guide, can also limit the results. These limits can be alleviated for symmetric wave paths, but only to the extent the wave guide can be shaped flat enough. I’ve shown that vibration noise is the next obstacle to future progress, and suggested that differential techniques be employed to overcome that obstacle. One second interferometer times have been shown here, with random phase. It is hoped that the correct differential technique could be used to make one second times useful.

More importantly this thesis presented demonstrations of how this instrument can be used to make measurements. There was discussion of gravity, rotation, optical and electric fields. All of these measurements were firsts for guided BEC interferometry. Of these the optical field measurements, were shown to be the best technique available. Much work will be needed to elevate the other three measurements to this level, but the work here can serve as a starting point in understanding how to attain precise measurements. It is important
to continue these efforts because measurements of gravity, rotation, and electric fields all have important applications for future measurement sensors. Electric field measurements can test fundamental theories of the atom and help make more accurate optical clocks for correspondingly better Global Positioning Satellites. Gyroscopes, along with clocks and gravitometers, can make inertial navigation systems that could be used for exploring areas of the planet that can’t access satellite signals such as underwater or cave environments.

6.2 Future Work

6.2.1 Polarizability

The polarizability experiment could find success if a new method of reaching the atoms with the standing wave can be found that does not suffer from diffraction effects. One solution might be to image a slit onto the field plates from both directions, with independent beams, but this will likely lead to unstable standing wave phase. Another solution would be to use non TEM$_{00}$ laser spatial modes to construct a flat-top intensity profile. Flat-top laser beams have been demonstrated in a Fabry-Perot cavity [94] that could be useful here. A flat-top beam would have the advantage of having uniform intensity over the BEC for the operations, but reduce to zero quickly to avoid diffraction from the field plates.

If no optical solution can be found, the field plates will need to be rebuilt with greater optical access. The atom separation must be much larger than the plate spacing so greater optical access will require larger atom separation than demonstrated here. A linear differential technique using the half-reflect operation described in Chapter 3 will need to be developed for this case. In that geometry, two independent interferometers operate simultaneously but displaced in the $y$ direction. In Figure 6.1, one can see how this could be made to work. The two-independent interferometers operate such that at their maximum separation, one packet in each interferometer is at the same location that the original BEC
Figure 6.1: Differential Polarizability Trajectory
A BEC starts in the center of the middle electric field region on the left, where it is split. When the two resulting packets are at the boundary of the middle field region, a half-reflect splits the two packets into four. Each pair forms an independent interferometer (dashed versus dotted) that operates symmetrically to the conductor boundaries indicated by the horizontal lines. Another half-reflect pulse recombines them and the four final states $|i, \pm\rangle$ can be measured.

was split. For each interferometer, the interference signal is that the number of atoms in the $|+\rangle$ states, $N^+$, will oscillate orthogonally to the $|\rangle$ states e.g. $N^+/N = \sin(\phi)/2$ and $N^-/N = \cos(\phi)/2$, where $N$ is the total atoms in all states.

The field plate that presently has two independent conductors would need three instead. The electric field could be applied to the atom wave-packets that are overlapped or to the wave packets that are maximally separated. In either case the phase due to the electric field will be opposite for the two interferometers while all other phases should be the same. Let us define the electric field phase $\phi_\alpha$ and all common phases $\eta$, which may be random. The sum of the atoms in the $|+\rangle$ states is

$$\frac{N_1^+ + N_2^+}{N} \equiv N_+ = \sin \eta \cos \phi_\alpha$$

(6.1)
A slanted geometry allows different $y$ spacing values to be used by using different $x$ positions.

and the $|−⟩$ states is

$$\frac{N_1^- + N_2^-}{N} \equiv N_- = \cos \eta \cos \phi_\alpha$$

(6.2)

With the sum of the squares of $N_+$ and $N_-$ we can recover $\phi_\alpha$ from a noisy phase

$$\frac{N_+^2 + N_-^2}{N^2} = \cos^2 \phi_\alpha$$

(6.3)

Another advantage to this geometry is that at the symmetry point in the middle electric field region, one could create a point where the field gradient went to zero. A disadvantage would be the conductor geometry would dictate the interferometer time and the atom wave packet spacing. To combat this disadvantage, a conductor geometry as in Figure 6.2 may be desirable because one could adjust the timing by choosing a different value of the $x$ position.

### 6.2.2 Rotation

The electric and rotation measurements both need a differential technique to be successful. An experiment is currently under construction designed specifically to measure rotation. This experiment will use a symmetric bowl shaped trap that will avoid the need of reflect pulses. Atoms placed at the center will be split in one direction. When they reach
6.2. FUTURE WORK

Figure 6.3: Differential Gyroscope
Atoms start in the center of the \( x - y \) diagram. They are first split in the \( x \) direction and then again in the \( y \) direction at the \( x \) classical turning point. Two pairs wave packets orbit in clockwise and counterclockwise directions before recombining with a third split pulse.

At the classical turning point they will be split again in a perpendicular direction. When everything is arranged correctly, the four wave atom packets will essentially be in orbit around the minimum of the central potential. One pair will be travelling clockwise while the other pair travels counterclockwise. After a full orbit the packets are recombined with another split pulse and the interference pattern observed. See Figure 6.3. In this geometry, most noise sources should cancel leaving only the Sagnac phase just as was shown above.

The Sagnac area would be \( A_s = (v_0t)^2 \) which for a one second time is 130 mm\(^2\). The corresponding rotation is for 10 mrad sensitivity is \( \Omega = 9 \times 10^{-11} \) rad/s, which is slightly better than the best atom gyroscope currently. With new advances in miniaturizing BEC devices and speeding up their production rate [26, 95] there is a possibility that a device to measure rotation accurately could be deployed in the field.
6.2.3 Gravity

There is potential improvement for the gravitational experiments discussed in Chapter 1 as well. This work does not use the guided wave technique described in this thesis, but it is another application of a BEC interferometer that we have studied (see the thesis of Jeramy Hughes [44]) and could be useful in the future. We attempted a vertical interferometer, but it worked poorly. With a two color standing wave, one can do much better. One also needs to work in the $m_F = 0$ magnetic sublevel so there won’t be any stray magnetic field effects. We can accumulate more phase by separating the atoms for longer times and separating them farther apart. The time in this case is set by the number of Bragg operations that can be made and the cycle time between operations. The cycle time is just $t_c = v_r/g$ for gravity, $g$. For an initial BEC of $N_0$ atoms, the number of reflection operations $j$ that can be achieved is set by the fidelity, $F$, of the operation and the number of atoms needed to make a measurement, $N_m$: $j = \ln \left( \frac{N_m}{N_0} \right) / \ln F$. For simple suspension the total time is:

$$T = 2jv_r/g = \frac{\ln N_m/N_0}{\ln F} \frac{h}{gm\lambda}$$  \hspace{1cm} (6.4)

Low mass, short wavelength and more atoms provide higher sensitivity. In the Introductory chapter, the demonstration used 15,000 Rubidium atoms at a mass of 87 amu and wavelength of 780 nm [42]. For simple suspension with $F = 0.999$ and the parameters above, a total suspension time of 8 seconds is possible with $10^4$ atoms remaining to make a measurement. Note that Lithium would be better because of its much lower mass.

An interferometer cannot use just the reflect pulse because we need two wave packets. In [42], we attempted an interferometer by introducing an asymmetry in the operations. One packet experienced reflections $|−1⟩ \leftrightarrow |+1⟩$, while the other experienced $|−2⟩ \leftrightarrow |+2⟩$ reflections as well. Unfortunately, this asymmetry in the reflect operations caused a phase imbalance between the two arms in addition to gravity. It was hard to separate the two affects which limited the measurement.
Figure 6.4: Ladder Interferometer
Atoms are bouncing until time $t = 0$ where they split into a packet moving upward at $v_r$ decelerating due to gravity (dashed curve) and a packet at rest accelerating downward (solid curve). At time $t_c$, a combination of momentum operations converts them back to the initial momentum state but now they are spatially separated. After time $jt_c/2$ they reverse, where in this diagram $j = 6$. Finally at $jt_c$ they recombine. After some time of flight (bold) the atom number in each state is measured.

A two color standing wave (one comprised of two counter-propagating laser beams of different frequencies) opens up new kinds of operations as demonstrated by [96]. With a similar setup the interferometer can be created with a sequence as in Fig. (6.4). Atoms at rest, $|0\rangle$ will be transferred to the state $(|0\rangle + |2\rangle)/\sqrt{2}$ creating a superposition of a packet moving upwards at $p = 2$ and one still at rest. At some time $t_c$ later, the upward moving component will be at rest, while the one at rest will be moving at $p = -2$ due to gravity. At that moment the transitions $|2\rangle \leftrightarrow |-2\rangle$ followed by $|0\rangle \leftrightarrow |2\rangle$ will be made which again makes the superposition of atoms moving up and at rest. This puts the atoms in the same initial momentum state, but the packets will be spatially separated by an amount $y_p = 4v_r^2/g$. Each iteration has the effect of incrementally separating the atomic wave packets. The “ladder” operation is reversed to bring the atoms together. When the packets overlap again, the original pulse is applied. Each packet will experience the same operations, so no extra phase develops. After recombination atoms will be in one of two states depending on the phase difference. The ratio of atoms in each state will be proportional to $\cos \Delta \phi$. The atoms can be counted with absorption imaging and the
6.2. FUTURE WORK

ratio will cancel any detection inefficiency. The shot noise measurement is established on
the number of atoms observed. The total phase difference will be:

\[ \phi_g = \frac{p^3 j^2}{4m^2 g^2 \hbar} \tag{6.5} \]

where \( j \) is the total number of ladder iterations, \( p \) is the recoil momentum given by an
operation, and \( T \) is the total time. In the description above \( p = 2\hbar k \), but could be as
high as \( 20\hbar k \) \[96\]. This scheme will develop phase cubically with momentum compared
to previous methods \[42, 10, 11\], where phase develops linearly with the momentum. To
measure the phase and thus gravity one simply needs to measure the ratio and know what
fringe the measurement was made on. This can be done by incrementing \( j \) as well as
applying a second known phase with the standing wave as in discussed in Chapter 1 by
changing the frequency of pulses slightly. If this was shot noise limited, then uncertainty
in \( g \) would be

\[ \delta g = \frac{1}{\partial \phi/\partial g} \frac{1}{\sqrt{N}} \tag{6.6} \]

The limit varies from \( 10^{-11} g \) for \( j = 100 \) as was shown in the Introduction chapter, to
\( 10^{-15} g \) for 10,000 pulses.

In reality, one will run into many hurdles before this precision could be reached. For
example, phase diffusion as discussed in Chapter 2 will limit this in an a fundamental way.
The resulting phase noise developed after time \( T \) is

\[ \phi_\mu = \frac{\mu T}{\hbar \sqrt{N}} = \frac{\mu p j}{h m g \sqrt{N}} \tag{6.7} \]

Once this is larger than \( \pi \), contrast will be significantly degraded. The sensitivities are
summarized in the table below. We have demonstrated \( \mu = 2\pi h \times 15 \) Hz, but it will be
hard to push much further than that. There is one other way to reach higher precision:
instead of using photon momentum of \( p = 2\hbar k \) as discussed, higher order operations could
be used instead. The phase sensitivity grows cubically with the momentum and so for
\( p = 6\hbar k \) operations, an order of magnitude is gained.

<table>
<thead>
<tr>
<th>Precision in units of ( g )</th>
</tr>
</thead>
<tbody>
<tr>
<td># operations, ( j )</td>
</tr>
<tr>
<td>shot noise limit</td>
</tr>
<tr>
<td>( \mu/2\pi h ) [Hz]</td>
</tr>
<tr>
<td>( \phi_\mu ) limit with ( p = 2\hbar k )</td>
</tr>
<tr>
<td>( \phi_\mu ) limit with ( p = 6\hbar k )</td>
</tr>
</tbody>
</table>

For comparison current atomic fountain techniques use packets of atoms falling several
meters in an interferometry arrangement. In [10, 11] sensitivities of the order \( 10^{-8} g \) to
\( 10^{-9} g \) are reported. There will of course be other technical limits limiting this measure-
ment, but a comparable precision as above may be possible in a tiny space compared to
the meter tall experiments.

There are many opportunities for BEC interferometry in the future. These are just
possible advances to few of the experiments discussed in this thesis.
Appendix A

Rubidium

Table A.1: Fundamental Constants

<table>
<thead>
<tr>
<th>Name</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Speed of light</td>
<td>$c$</td>
<td>$2.99792458 \times 10^8$ m/s</td>
</tr>
<tr>
<td>Standard gravity</td>
<td>$g$</td>
<td>9.80665 m/s</td>
</tr>
<tr>
<td>Permeability of vacuum</td>
<td>$\mu_0$</td>
<td>$4\pi \times 10^{-7}$ N/A²</td>
</tr>
<tr>
<td>Permittivity of vacuum</td>
<td>$\epsilon_0$</td>
<td>$8.854187817 \times 10^{-12}$ F/m</td>
</tr>
<tr>
<td>Planck’s constant</td>
<td>$h$</td>
<td>$6.62606876 \times 10^{-34}$ Js</td>
</tr>
<tr>
<td></td>
<td>$\hbar = h/2\pi$</td>
<td>$1.054571596 \times 10^{-34}$ Js</td>
</tr>
<tr>
<td>Elementary charge</td>
<td>$e$</td>
<td>$1.602176462 \times 10^{-19}$ C</td>
</tr>
<tr>
<td>Bohr magneton</td>
<td>$\mu_B$</td>
<td>$9.27400899 \times 10^{-24}$ J/T</td>
</tr>
<tr>
<td>Atomic mass unit</td>
<td>$u$</td>
<td>$1.66053873 \times 10^{-27}$ kg</td>
</tr>
<tr>
<td>Electron mass</td>
<td>$m_e$</td>
<td>$9.10938188 \times 10^{-31}$ kg</td>
</tr>
<tr>
<td>Bohr radius</td>
<td>$a_0$</td>
<td>$0.5291772083 \times 10^{-10}$ m</td>
</tr>
<tr>
<td>Boltzmann’s constant</td>
<td>$k_B$</td>
<td>$1.3806503 \times 10^{-23}$ J/K</td>
</tr>
</tbody>
</table>
Table A.2:
Properties of $^{87}$

<table>
<thead>
<tr>
<th>Name</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Atomic mass</td>
<td>$m$</td>
<td>$86.909,180,520$ u</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$1.44316060 \times 10^{-25}$ kg</td>
</tr>
<tr>
<td>Nuclear spin</td>
<td>$I$</td>
<td>$3/2$</td>
</tr>
<tr>
<td>Scattering length</td>
<td>$a$</td>
<td>$5.77$ nm</td>
</tr>
</tbody>
</table>

$^{5\,2}S_{1/2} \rightarrow ^{5\,2}P_{3/2}$ Transition properties

<table>
<thead>
<tr>
<th>Name</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency</td>
<td>$\omega_o$</td>
<td>$2\pi \cdot 384.2304844685$ THz</td>
</tr>
<tr>
<td>Wavelength (vacuum)</td>
<td>$\lambda$</td>
<td>$780.241209686$ nm</td>
</tr>
<tr>
<td>Lifetime</td>
<td>$\tau$</td>
<td>$26.24$ ns</td>
</tr>
<tr>
<td>Natural linewidth</td>
<td>$\Gamma$</td>
<td>$2\pi \cdot 6.065$ MHz</td>
</tr>
<tr>
<td>Recoil velocity</td>
<td>$v_r$</td>
<td>$5.8845$ mm/s</td>
</tr>
<tr>
<td>Recoil temperature</td>
<td>$T_r$</td>
<td>$361.96$ nK</td>
</tr>
<tr>
<td>Doppler temperature</td>
<td>$T_D$</td>
<td>$146$ $\mu$K</td>
</tr>
<tr>
<td>Electron spin $g$-factor</td>
<td>$g_S$</td>
<td>$2.0023193043737$</td>
</tr>
<tr>
<td>Electron orbital $g$-factor</td>
<td>$g_L$</td>
<td>$0.99999369$</td>
</tr>
<tr>
<td>$g_J(5,^2S_{1/2})$</td>
<td>$g_J(5,^2P_{1/2})$</td>
<td>$2.00233113$</td>
</tr>
<tr>
<td>Landé $g$-factor</td>
<td>$g_J(5,^2P_{3/2})$</td>
<td>$0.666$</td>
</tr>
<tr>
<td>Nuclear $g$-factor</td>
<td>$g_I$</td>
<td>$-0.0009951414$</td>
</tr>
</tbody>
</table>
Figure A.1: $^{87}$Rb principal transitions with hyperfine structure. Light at 780.246 nm is used for the MOT transitions, whereas 780.232 nm light is used for “repumping” atoms that have fallen into the $F = 1$ ground state.
Appendix B

Cavity Transfer Lock

B.1 Paper Published in Rev. Sci. Inst.

Many physics experiments require the frequency of a tunable laser to be held at a desired value. The most common way to achieve this is to lock the laser to an atomic or molecular resonance, often using saturated absorption techniques to obtain a narrow linewidth and correspondingly better frequency resolution.[97] Although effective, this technique has some disadvantages. For instance, it requires the desired frequency to be close to a convenient reference transition, which is not always possible. Also, the capture range of a saturated absorption lock is small, typically a few tens of MHz in frequency. This makes the system susceptible to coming unlocked in response to a perturbation and also makes the system difficult to relock automatically. This can be an annoyance for laboratory work or a serious problem for applications where user intervention is not possible.

One alternative technique is the scanning transfer cavity lock. [98, 99, 100, 101, 102] Rather than using an absolute reference, this method stabilizes the frequency difference between two lasers by passing beams from both lasers through a Fabry-Perot cavity. The mirror separation of the cavity is repeatedly scanned across one or more free spectral ranges (FSRs). As the separation changes, one laser eventually comes into resonance,
resulting in a pulse of transmitted light. At a later time, the second laser comes into resonance. The lock works by determining the difference between the two pulses and controlling the frequency of the target laser to hold the difference constant. If one of the lasers is locked to an absolute reference, this stability can be transferred to the second laser. The second laser can be at any frequency and the lock’s capture range is set by the cavity FSR, typically a few GHz. The transfer lock thus overcomes many of the problems of the saturated absorption lock.

The primary disadvantages of the transfer lock have been its expense and complexity. It requires a stable reference laser, a transfer cavity, and a control system. Because of this, it has been more common to use a conventional saturated absorption lock unless the special capabilities of the transfer lock are essential. However, in many experiments, a reference laser is already available. For instance, laser cooling experiments require one laser tuned near an atomic resonance to provide the cooling. If this laser is locked via saturated absorption, it can serve as the reference for any other lasers the experiment uses.

Other implementations of a transfer lock that we are familiar with use a dedicated, stabilized optical cavity. Typically, the two output pulses are separated using polarization or wavelength filters, though Rossi et al. detected the combined beams with a single photodiode.[101] Early designs used electronic control systems,[98, 99] but more recent versions use a personal computer equipped with a digital-to-analog converter (DAC).

We believe that the transfer lock described here is the simplest and most cost-effective version demonstrated to date. For our cavity, we use an unmodified commercial spectrum analyzer system. In our experience, many labs already use a spectrum analyzer for other purposes. For instance, it is the best way to verify that diode lasers are operating in a single mode. Our control system is based on a microcontroller card that, once programmed, does not require a separate computer system. The entire control system costs less than US$200 and fits on a 2-inch rack panel. We believe that in many cases, this system can be a cheaper and better replacement for a saturated absorption lock.
Figure B.1: Schematic of lock system
Beams from a stabilized reference laser and a tunable laser are sent into a spectrum analyzer. The upper graph shows a typical output signal. The conditioning circuit of Fig. 2 converts this to a digital signal as shown. The microcontroller inputs the signal, calculates the feedback voltage, and applies it to the tunable laser via a digital-to-analog converter.

Figure B.1 shows a schematic of the servo loop for the lock. Our reference laser is a Coherent MBR Ti:Sapphire laser locked to the $F = 2 \leftrightarrow F' = 3$ transition in $^{87}\text{Rb}$ at 780 nm. The tunable laser is a homebuilt external cavity diode laser based on a Sanyo DL7140-201S, 80 mW, 785 nm laser diode. The laser is driven by a ThorLabs ITC502 controller and the cavity grating is positioned with a ThorLabs MDT694A piezoelectric controller. Beams from each laser are combined on a beam splitter and directed into the spectrum analyzer. About 200 $\mu$W of power from each laser is used.
The spectrum analyzer is a Coherent Model 251, which is a scanning confocal cavity with a FSR of 1.5 GHz. It has a nominal finesse of 550 over the wavelength range 690–830 nm. The analyzer scans over two FSRs with a 20 ms period. The upper graph in the figure shows a typical output display. The lock operates by measuring the times $t_1$ and $t_2$ and adjusting the feedback to keep $t_1/t_2$ constant. By controlling the ratio, variations in the scan rate cancel out.

The time measurements and feedback calculations are made using an IsoPod V2-SR1 microcontroller card from New Micros, Inc. The IsoPod has a function that measures digital pulse durations, so we use a signal conditioning circuit to convert the timing information from the spectrum analyzer to logical pulses as shown. The signal conditioning circuit is shown in Fig. B.2. Noise on the input signal causes the conditioning circuit to make a spurious transition about once every 500 scans. This is detected and handled by the microcontroller.

![Figure B.2: Conditioning circuit](image)

The spectrum analyzer signal is amplified, filtered, and differentiated to produce a signal that crosses zero at the peak of the pulse. A comparator is used to locate this zero crossing. Since the differentiated signal is noisy when no pulse is present, it is gated with a second comparator that is high only when the original signal exceeds a fixed threshold, typically about 50 mV. The resulting pulses trigger a flip-flop to produce a digital signal that changes state at the peak of each pulse. The S input to the flip-flop is derived from the trigger output of the spectrum analyzer. The trigger is low during the forward scan of the cavity and high during the backward scan. The analyzer output is zeroed during the backward scan to avoid hysteresis, so we suppress the pulse sequence during that time.

The microcontroller is a state machine, which allows it to run in parallel several pro-
grams simultaneously without need for multiple on-board CPUs or a multitask-oriented operating system. We do not effectively use this ability because we only need to lock one laser, but if required the microcontroller should be able to manage several locks at once.\cite{100} A state machine works as follows: for any state there are some number of transitions to and from the state. Each transition has a condition which when met requires the machine to move to the new state and execute any programming defined in the transition. The machine continuously checks the conditions for exiting the current state.

The controller uses a language derived from FORTH. The program definitions are uploaded to the device from a desktop PC through a serial connection. Once loaded, they are stored in persistent memory and run automatically when the controller is turned on. Since state machines may be less familiar than ordinary computers, we explain the structure of our program below. A copy of the program can be obtained from the authors on request.

Figure B.3: Lock State diagram
Circles represent states and arrows represent transitions. Transitions are labeled with their conditionals and commands.

Figure ?? is a state diagram for our program. The system begins in the Rest state.
The Lock On condition is set by a switch on the front panel and the Hold Off condition is an external input. When the lock is turned on, the Reset Lock routine measures $t_1$ and $t_2$ using two successive scans from the spectrum analyzer. The timing resolution of the counter is 0.4 $\mu$s. For our scan rate and FSR, this corresponds to a frequency resolution of 120 kHz. The controller has two routines to reject signal conditioning errors. Most errors from the circuit are double triggers where the circuit produces two transitions under one spectrum analyzer pulse. The program tests for and ignores any ratio that correspond to this type of error. Further, each ratio measured must agree with the previous two within 10%. Once three good measurements are obtained, they are averaged and assigned as the set point for the lock.

Most of the activity takes place in the Run state. During the Run Lock routine, the controller first measures $t_1$ and $t_2$ as above and checks for errors. Once a good value is obtained, the error signal is computed as the difference between the ratio and the set point. Integral feedback is implemented by adding the error signal to a running sum, which is then multiplied by a gain factor and output to the DAC. We used the MAX507, a standard 12-bit converter. A voltage divider reduces the analog signal by a factor of 3 before it is applied to the external input of the piezoelectric controller.

The Hold state is not needed for the lock, but we required the capability to unlock the laser so that it could be externally scanned to a different frequency during an experiment and then tuned back to the original frequency and relocked when the experiment is complete. This permits us to use the laser for two purposes during the experiment. The Hold On/Off conditions are set by an externally applied logic signal that comes from the main control system for the experiment. In the Hold state, the IsoPod maintains its current output level and set point, but does not make any new measurements. The ease with which a capability such as this can be added illustrates the versatility of a computer-controlled system.

We evaluated the performance of the transfer lock using three tests. First, we locked
the diode laser to the $F = 1 \leftrightarrow F' = 2$ transition of $^{87}$Rb and used it as the repump laser for a magneto-optical trap (MOT). The MOT performance was no different than observed using a saturated absorption lock. The diode laser generally lost its lock only when the reference laser itself came unlocked. A second test used a heterodyne measurement with the reference laser. We locked the two lasers about 300 MHz apart and directed the combined beams from the lasers onto a photodiode. The observed beat note frequency fluctuated by about 1 MHz on a 1 s timescale and by about 0.3 MHz on a 1 hour time scale. For the third test, we ran the lock in an open loop mode. The diode laser was locked via saturated absorption to the $^{87}$Rb $F = 1 \leftrightarrow F' = 2$ transition and the error signal from the transfer lock was monitored. The microcontroller program was modified so that instead of outputting a running sum of the error signal, a running average was calculated with a 2.4 s time constant. Over a 1 hour time scale, the observed voltage fluctuations corresponded to frequency noise of less 300 kHz.

These tests indicate that the transfer lock controlled the long-term stability of the laser frequency to better than 1 MHz accuracy. This is comparable to the performance achieved with previous systems. It should be noted that the tunable laser is required to have good short-term stability, since the transfer lock cannot provide feedback faster than the 25 Hz data acquisition rate. Faster data rates have been implemented in Refs. Jaffe93 and Matsubara05.

Our simple implementation has several limitations. Because we detect both lasers with a single photodiode, the lock will not work if the frequency difference is close to an integer number of FSRs. This could be overcome by separating the transmitted beams and using two detectors. The lock is also sensitive to drifts in the optical cavity length, in two ways. First, a change in the FSR can cause the target laser frequency to vary if the detuning between the two lasers, $\Delta$, is large. Typically, the FSR of our cavity drifts at a rate of
$R \approx 1 \text{ kHz/hour}$. The resulting drift in the target laser frequency is

$$\dot{\nu} = \frac{\Delta}{\text{FSR}} R.$$  \hspace{1cm} (B.1)

In our case, $\dot{\nu}$ is less than 1 MHz/hour for $\Delta < 1.5 \text{ THz}$, or about 3 nm at 780 nm. Locking two lasers with considerably different wavelengths would therefore require a stabilized cavity. For smaller detunings (notably including atomic hyperfine splittings), this effect is negligible.

Even for small detunings, cavity drift causes the peak locations in the output trace to shift. With reference to Fig. 1, eventually one of the peaks will drift off the display and a new peak of neighboring order will reappear on the other side. When this occurs, the microcontroller no longer measures the correct times and the lock fails. To prevent this, it is necessary to adjust the offset control of the spectrum analyzer every few hours. If this were not possible, a more sophisticated program could be used to detect the change in timing and automatically compensate for it.

In summary, we have demonstrated a simple version of a scanning transfer cavity lock. It was able to successfully stabilize a target laser’s frequency to better than 1 MHz over long time scales. Given that a reference laser and a spectrum analyzer were already available, the lock system was inexpensive and required no space on the optical table. Further benefits include the ability to lock a laser at arbitrary frequencies and to permit complex lock control via software. We hope that this work will make it easier to take advantage of the transfer technique.

**B.2 Program Code**

( Cavity Transfer Lock )

( program and state machine )

( John H T Burke, 2005 )
( to install: select all of this txt, copy )
( and paste into "NMIterm" terminal window for Isopod )
( stack notes: a,b---c,d means before operation a,b on stack )
( and after c, d are on stack. F: indicates float stack )
( flag note: 0 is false not 0 is true )

SCRUB  ( erases ROM )
FVARIABLE T_HIGH  EEWORD  ( declare all variables make room in ROM )
FVARIABLE T_LOW  EEWORD
FVARIABLE R1  EEWORD
FVARIABLE R2  EEWORD
FVARIABLE R3  EEWORD
FVARIABLE GAIN_SIGN  EEWORD
FVARIABLE R_SET  EEWORD
FVARIABLE V_OUT  EEWORD
FVARIABLE N  EEWORD
FVARIABLE V_ERR  EEWORD
FVARIABLE GAIN  EEWORD
FVARIABLE GLITCH_TOLERANCE  EEWORD
FVARIABLE R_MIN  EEWORD
FVARIABLE WGHT  EEWORD
VARIABLE R_INDEX  EEWORD

: MEASURE-PWM  ( F: ---R )
( Measures pulse times )
( STORES T_HIGH AND T_LOW )
( CALCULATES THE RATIO AND STORES IT )
( BASED ON R_INDEX )

0.0E0
BEGIN
FDROP  ( DROPS THE 0 ABOVE OR A BAD RATIO IF THIS LOOPS )
BEGIN  ( 2 loops to cycle trigger )
PA0 OFF?  ( ---truth flag for state of pin PA0 )
UNTIL  ( flag--- )
BEGIN
PA0 ON?
UNTIL
TD1 ACTIVE-HIGH  ( set to measure high side of pulse )
TD1 SET-PWM-IN
BEGIN
TD1 CHK-PWM-IN  ( ---T which could be 0 if no pulse yet )
?DUP  ( IF T NOT = to 0, copys the time[--T,T] )
( ELSE does nothing )
UNTIL
  ( WHEN T NOT 0 [TRUE FLAG] UNTIL EXITS )
  ( leaving ---T )
S>F
  ( T--- F: ---T converts time to float )
T_HIGH F!
  ( F:T--- STORE T_HIGH )
BEGIN
  ( same as above for the low side )
  PA0 OFF?
UNTIL
BEGIN
  PAO ON?
UNTIL

TD1 ACTIVE-LOW
TD1 SET-PWM-IN
BEGIN
  TD1 CHK-PWM-IN
  ?DUP
  UNTIL
S>F T_LOW F!
T_LOW F@ FDUP T_HIGH F@  ( F: ---T_LOW,T_LOW,T_HIGH )
F+ F/  ( F: T_LOW,T_HIGH+T_LOW---R )
  ( IF RATIO IS LESS THAN R_MIN TRY AGAIN )
R_MIN F@ FOVER F<  ( F: R,R_MIN,R---R and ---FLAG )
UNTIL
R_INDEX @ 1 = IF
  R1 F!  ( F: R--- STORE IN R1 IF R_INDEX = 1 )
  R_INDEX @ 1 + R_INDEX !  ( INCREMENT INDEX )
ELSE
  R_INDEX @ 2 = IF  ( store in R2 for R_index =2 )
    R2 F!
    R_INDEX @ 1 + R_INDEX !
ELSE  ( store in R3 and reset index to 1 )
  1 R_INDEX !
  R3 F!
THEN
THEN
; EEWORD

: GLITCH  ( ---FLAG  ( checks to see if R1, R2, and R3 are within some )
  ( percent of each other given by Glitch_tolerance )
R2 F@ R1 F@  ( F: ---R2,R1 )
F- R2 F@  ( F: R2,R1---[R2-R1],R2 )
F/ FABS  ( F: [R2-R1],R2---[R2-R1]/R2[ ] )
GLITCH_TOLERANCE F@ F<  ( F: [R2-R1]/R2,GLITCH_TOLERANCE]--- [---FLAG ]
R2 F@ R3 F@ F- R2 F@ F/ FABS GLITCH_TOLERANCE F@ F<  ( ---FLAG,FLAG )
R1 F= R2 F= R3 F= R1 F= R2 F= R3 F= FABS GLITCH_TOLERANCE F= F< ( ---FLAG,FLAG,FLAG )
AND AND ( [FLAG,FLAG,FLAG---FLAG] ONLY TRUE IF ALL FLAGS ARE TRUE )
; EEWORD

: SET_POINT ( F:--- STORES SET VALUE as the average of three ratios )
BEGIN
3 0 DO
  MEASURE-PWM ( F:---R1,R2,R3--- MEASURES and stores )
LOOP
  GLITCH
UNTIL
R1 F= R2 F= R3 F= R1 F= R2 F= R3 F= 3.0E0
F/ R_SET F! ( F: SUM123,3--AVG-- STORE IN R_SET )
; EEWORD

: ERRCALC ( --- STORES V_ERR )
BEGIN
  PE4 OFF PE5 OFF PE6 OFF
  MEASURE-PWM ( F: ---RATIO )
  GLITCH
UNTIL
R_INDEX = 1 = IF ( THE INDEX HAS ALREADY ADVANCED )
  R3 F= R_SET F= V_ERR F! ( F: RATIO,SET---V_ERR--- )
THEN
R_INDEX = 2 = IF
  R1 F= R_SET F= V_ERR F!
THEN
R_INDEX = 3 = IF
  R2 F= R_SET F= V_ERR F!
THEN
; EEWORD

: SUM ( --- CONTINUES SUM )
BEGIN
  N F= WGHT F= F* ( F: SUM,WGHT---WGHTD SUM
  V_ERR F= F+ N F! ( F: WGHTD_SUM,V_ERR---NEW_SUM--- )
END

: OUTPUT ( --- WRITES 11 BITs to DAC )
( see below for 1st sign bit)
1024.0E0 V_OUT F= F<
  IF PB3 ON V_OUT F= 1024.0E0 F- V_OUT F!
  ELSE PB3 OFF
  THEN
512.0E0 V_OUT F= F<
IF PB4 ON V_OUT FØ 512.0E0 F- V_OUT F!
ELSE PB4 OFF
THEN
256.0E0 V_OUT FØ F<
IF PB5 ON V_OUT FØ 256.0E0 F- V_OUT F!
ELSE PB5 OFF
THEN
128.0E0 V_OUT FØ F<
IF PB6 ON V_OUT FØ 128.0E0 F- V_OUT F!
ELSE PB6 OFF
THEN
64.0E0 V_OUT FØ F<
IF PB7 ON V_OUT FØ 64.0E0 F- V_OUT F!
ELSE PB7 OFF
THEN
32.0E0 V_OUT FØ F<
IF PA7 ON V_OUT FØ 32.0E0 F- V_OUT F!
ELSE PA7 OFF
THEN
16.0E0 V_OUT FØ F<
IF PA6 ON V_OUT FØ 16.0E0 F- V_OUT F!
ELSE PA6 OFF
THEN
8.0E0 V_OUT FØ F<
IF PA5 ON V_OUT FØ 8.0E0 F- V_OUT F!
ELSE PA5 OFF
THEN
4.0E0 V_OUT FØ F<
IF PA4 ON V_OUT FØ 4.0E0 F- V_OUT F!
ELSE PA4 OFF
THEN
2.0E0 V_OUT FØ F<
IF PA3 ON V_OUT FØ 2.0E0 F- V_OUT F!
ELSE PA3 OFF
THEN
1.0E0 V_OUT FØ F<
IF PA2 ON
ELSE PA2 OFF
THEN
; EEWORD

: OUTLOGIC ( DAC takes offset 2’s complement so this converts )
( to that format and writes 1st sign bit )
V_OUT F! ( F: N--- STORES SUM IN V_OUT )
V_OUT F@ FNEGATE F0< ( F: -V_OUT--- [---FLAG] )
IF ( FLAG--- )
   PB2 ON
   OUTPUT
THEN
V_OUT F@ F0<
IF
   PB2 OFF
   V_OUT F@ 2048.0E0 F+ V_OUT F! ( offset adjustment for neg #'s )
   OUTPUT
THEN
V_OUT F@ F0= IF
   PB2 ON PB3 OFF PB4 OFF PB5 OFF PB6 OFF PB7 OFF
   PA7 OFF PA6 OFF PA5 OFF PA4 OFF PA3 OFF PA2 OFF
THEN
   PWMA3 OFF
   PWMA4 OFF
   PWMA4 ON
   PWMA3 ON
   PWMA5 OFF
   PWMA5 ON
; EEWORD

; MAIN
   ERRCALC ( --- )
   SUM ( --- )
   N F@ GAIN F@ F* GAIN_SIGN F@ F* OUTLOGIC ( N--- )
; EEWORD

; SWITCH_CHECK_OFF ( double checks if the switch is off. )
   ( momentary voltage drop won't affect lock )
PA1 OFF? IF
BEGIN
   PA0 OFF?
UNTIL
BEGIN
   PA0 ON?
UNTIL
   PA1 OFF?
ELSE
   0
THEN
; EEWORD
: TESTSIGN  ( CHECKS SIGN FOR POSTIVE GAIN )
  1 R_INDEX !
  0.0E0 OUTLOGIC
  BEGIN ( cycle trigger )
    PA0 OFF?
    UNTIL
    BEGIN
      PA0 ON?
      UNTIL
    MEASURE-PWM ( MEASURE R1 )
  50.0E0 OUTLOGIC
  BEGIN
    PA0 OFF?
    UNTIL
    BEGIN
      PA0 ON?
      UNTIL
    MEASURE-PWM ( MEASURE R2 )
  0.0E0 OUTLOGIC
  R1 F0 R2 F0 F-
  F0< IF
    -1.0E0 GAIN_SIGN F!
  ELSE
    1.0E0 GAIN_SIGN F!
  THEN
 ; EEWORD
MACHINE DIODELOCK EEWORD
ON-MACHINE DIODELOCK
APPEND-STATE RUN EEWORD
APPEND-STATE REST EEWORD
APPEND-STATE HOLDING EEWORD
IN-STATE
  RUN
  CONDITION
    SWITCH_CHECK_OFF
  CAUSES
    REDLED ON
    GRNLED OFF
B.2. PROGRAM CODE

YELLED OFF
PWMB3 ON
PWMB1 OFF
PWMB2 OFF
0.0E0 OUTLOGIC

THEN-STATE
REST
TO-HAPPEN
IN-EE

IN-STATE
RUN
CONDITION
PA1 ON? PB0 OFF? AND
CAUSES
REDLED OFF
YELLED OFF
GRNLED ON
PWMB1 ON
PWMB2 OFF
PWMB3 OFF
MAIN
REDLED ON
THEN-STATE
RUN
TO-HAPPEN
IN-EE

IN-STATE
RUN
CONDITION
PB0 ON? PA1 ON? AND
CAUSES
REDLED OFF
YELLED ON
GRNLED OFF
PWMB3 OFF
PWMB2 ON
PWMB1 OFF
THEN-STATE
HOLDING
TO-HAPPEN
IN-EE
IN-STATE
  REST
CONDITION
  PA1 ON?
CAUSES
  0.0E0 N F!
  SET_POINT
  TESTSIGN
  REDLED OFF
  GRNLED ON
  PWMB1 ON
  PWMB3 OFF
THEN-STATE
  RUN
TO-HAPPEN
IN-EE

IN-STATE
  HOLDING
CONDITION
  PB0 OFF? PA1 ON? AND
CAUSES
  YELLED OFF
  GRNLED ON
  PWMB2 OFF
  PWMB1 ON
THEN-STATE
  RUN
TO-HAPPEN
IN-EE
IN-STATE
  HOLDING
CONDITION
  SWITCH_CHECK_OFF
CAUSES
  YELLED OFF
  REDLED ON
  PWMB2 OFF
  PWMB3 ON
  0.0E0 OUTLOGIC
THEN-STATE
  REST
TO-HAPPEN
IN-EE
B.3 Program Explained

See the ISOPOD manual section 7 for a glossary of all words used in the program that aren't explained here.

State Machine has 3 states: REST, RUN, and HOLD. Transitions between states are controlled by the switch on front or BNC. Down is off, Middle is Lock on and Hold, top is Lock on with BNC control of HOLD.

The LED's on the ISOPOD indicate the state as follows: RED on, GREEN and YELLOW off equal REST. Yellow on, GREEN and RED off equal HOLD. GREEN on, YELLOW off, RED blinking equal RUN. Note that the RED blinks each time the the lock program cycles.

ON Powering up, the ISOPOD starts the state machine in the initial state of REST, zeroes the output and initializes variables with the following default values:

WGHT = 1.0
GAIN = 1000
GLITCH_TOLERANCE = 0.1 ( 10% )
R_MIN = 0.01
R_INDEX = 1
R1 = R2 = R3 = 0
these will be explained later.
To view or change any of these values temporarily, at the terminal, write
"variable" F@ F. to print the value
"value" "variable" F! to store value in variable.
to store permanently they must be changed in the STARTUP function and the entire
program reloaded.
The 'heart' of this program lies in the function MEASURE-PWM.
-This waits on trigger cycle measures the high time, then waits another trigger cycle
and measures the low time.
-It then measures the ratio of the low time over the sum of the low and high, which
should be 1 free spectral range (1.5 GHz).
-It checks if the ratio is greater than R_MIN. R_MIN would only happen if the two
peaks were just barely resolved or worse, which means the conditioning probably had a
double trigger (when there is more than one flip flop transition for a given peak.)
-It stores the ratio (if its bigger than R_MIN) in R1, R2, or R3 depending on the
R_INDEX, which it increments after each successful measurement.
Transition from REST to RUN:
If the switch goes to the middle or high postion the machine transitions to RUN
(regardless of the HOLD status). In the transition, the program zeros the Running sum
stored in variable "N" and calls the following functions to prepare the lock.
TESTSIGN
This function measures a ratio, outputs a small voltage, then measures another ratio.
The sign of the difference of these ratios is the sign of gain to use for negative feedback.
SET_POINT
This measures three ratios then calls GLITCH (see below) before averaging the ratios
to create the reference to lock the laser. This is stored in R_SET
GLITCH:
Checks if the values of R1, R2 and R3 are within some percent of eachother, given by
'GLITCH TOLERANCE'
RUN cycle transition:
The program runs MAIN, which is comprised of the following:
ERRCALC:
Measures a ratio, checks with GLITCH, then computes the difference of the Ratio with
R_SET. This error is stored in V_ERR.
SUM:
Integrates the latest V_ERR into a running sum stored in 'N'. If desired a weight can
be used to make the sum into a weighted average, by changing the variable WGHT from
1.0.
The sum is then multiplied by GAIN and GAIN_SIGN and output with
OUTLOGIC:
The DAC takes a 2's complement offset binary number its input. The number on the
float stack before this is called is stored in V_OUT for manipulation. The function detects
the sign of V_OUT and adjusts the 12th bit (PIN PB2 ) to correspond. For positive
numbers the binary is the same as normal, but for negative its offset from normal so all zeros is -2048. The function adds 2048 to the value and outputs so its in the correct format for the function OUTPUT, which can then also be used for positive numbers. Note zero is 1000 0000 000 NOT 0000 0000 0000 which is -2048 as above and zero in floating point will display in decimal as some very large number.

OUTPUT:
Just writes the remaining 11 bits of output to pins PA2...PA7,PB7...PB3 with PA2 being the 1st bit.

From RUN, if the switch goes to off, the machine transitions to REST again and Zeros the output. Note that this transitions condition is actually the function SWITCH_CHECK_OFF, which just leaves a true value if the switch goes off, and is still off when the trigger cycles once, which protects against momentary voltage drops.

From RUN if the switch is in either the middle position or +5 V comes from the BNC in the high position, the machine will transition to the state HOLDING. In this state nothing happens. It just waits for the switch to change, or the BNC to be ground.

IN HOLD, the machine could transition to REST if SWITCH_CHECK_OFF is true or to RUN again if the HOLD is off and the the lock is on.

UPDATE 5.0 2/5/2007
A switch was added to allow the Hold to come in from the computer to work properly. An adder was placed to mix an analog input from the computer to the output of the DAC. It also sends a voltage to the hold if a pulse comes in on the input.

To the program was added a portion to control the LDAC, CS, and WR components of the DAC. When CS and WR go low data form the inputs is loaded into the DAC but not output as a voltage. When LDAC goes low the data loaded is output as a voltage. The ISOPOD pins that control these functions are: CS is pin PWMA3, WR to PWMA4 and LDAC to PWMA5. The controls are at the end of OUTLOGIC function.

Also LED’s were placed on the front panel to indicate the state and progress of the program. They are turned as follows: PWMB1 is green, PWMB2 is yellow, and PWMB3 is red. They are turned on in the state declarations.

Also for the record HOLD is connected to pin PB0 and lock on in connected to pin PA1. The trigger goes to PA0 and the digitized spectrum analyzer signal is on TD1 a timer pin.
Appendix C

3d simulation code

On the following pages is Matlab code to run a simulation of an interferometer working in three dimensions.

The function full3d_int.m takes the pulse time “tau” (1/4 of the total interferometer time), an oscillation amplitude three vector “Amp” and time offset three vector “t0” to define the initial conditions, and a laser phase “chi” as arguments and returns the following arrays: X, P, and b contain the position, momentum and squared amplitude information respectively of all of the wave packets created by the Bragg operations. They are indexed as indicated in the code comments on line 62 and 74. The value of N is meant to be versatile and can return the fraction of atoms that come back to rest, or an array of fractions of atoms in multiple states if one wants to calculate both the rest fraction and an asymmetry. Several possibilities are commented in lines 113 through 121. This function calls several attached sub-functions to compute rotations, project between harmonic oscillator and plane wave bases, and Bragg beam operations as described in Chapter 4.

The script Interferometer3dho_Main uses the above functions to simulate an experiment by looping over various input variables, fitting output data, then calculating experimental observables such as the visibility.
global M
%tau=5e-3; % time between split and reflect, T=4tau
M = 87.0/6.022e26; %mass of Rb in kg
%the applied phase is chi
ciarray=[0:0:6:6.6]*pi;
tauarray=[5:5:10]*1e-3;

Amp=zeros(1,3);
%Amp(1)=0;
Amp(2)=20/190/1000;

Amp(3)=20/190/1000;

Aranger=[0:50:300]/190*1e-3; % amplitude to vary
%Arange=[76 153 255]/190*1e-3;
%Amp(1)=150/190/1000;
iter=1;

Visibility=zeros(length(tauarray),length(Aranger),iter);
y0=zeros(length(tauarray),length(Aranger),iter);
V0=zeros(length(tauarray),length(Aranger));
Splitspeed=zeros(length(Aranger)*length(tauarray),1);
VJ=zeros(length(splitspeed),1);
V0J=zeros(length(splitspeed),1);

for u=1:iter %number of visibility curves to take, average over them to get data
    J=1;
    for s=1:length(Aranger) % range of oscillation amp to consider
        for t=1:length(tauarray)
            tau=tauarray(t);
            %tau=10e-3;
            %sets initial amplitudes

            Amp(1)=Aranger(s);
            Splitspeed(J)=Amp(1)*2*pi*5.6*sin(2*tau*2*pi*5.6);

            Aj(3)=Aranger(s);
            %tau=tauarray(s);

            AsigX=A.1*Amp(1); %error in amplitude from shot to shot
            AsigY=A.1*Amp(2);
            AsigZ=A.1*Amp(3);

            Asize=4; %get 4 amplitudes to take data for a given chi
            Amp_inX = normrnd(Amp(1),AsigX,1,Asize); %these are the amps to use
            Amp_inY = normrnd(Amp(2),AsigY,1,Asize); %a set of 3 random amps with normal distribution of standard deviation
            Asig
            Amp_inZ = normrnd(Amp(3),AsigZ,1,Asize);

            N8Nrun=zeros(length(choarray)*Asize,2); %a run of N8/N data

            Nout=zeros(3,length(Asize),length(choarray));
            %Nzero=zeros(length(Amp_in),length(choarray));
            %Nminu=zeros(length(Amp_in),length(choarray));
            Nasy=zeros(length(Asize),length(choarray));
            bout=zeros(3,3,3,3,4);
            cout=zeros(3,3,3,3,4); %dummy to hold off phase info
            Xout=zeros(3,3,3,3,4);
Pout=zeros(3,3,3,3,3,4);

%Mpin(2)=Amp(2);
%Mpin(3)=Amp(3);
L=1;
for n=1:length(chiarray)
    Amp_inX = normrnd(Amp(1),AsigX,1,Asize);
    Amp_inY = normrnd(Amp(2),AsigY,1,Asize);
    Amp_inZ = normrnd(Amp(3),AsigZ,1,Asize);
    for m=1:Asize
        chi=chiarray(n);
        Ampin(1)=Amp_inX(m);
        Ampin(2)=Amp_inY(m);
        Ampin(3)=Amp_inZ(m);
        if n==1

            [N0Nrun(1,1),Nasy(m,n),Nout(1,m,1),Nout(2,m,1),Nout(3,m,1),Xout
            t,Pout,bout]=full3d_int(tau,chi,Ampin); %just the zero phase
                end
            else
                [N0Nrun(1,1),Nasy(m,n),Nout(1,m,1),Nout(2,m,1),Nout(3,m,n),c
                out, cout, cout(1,:,:),cout(2,:,:)]=full3d_int(tau,chi,Ampin);
                end
            end

    end

    cosfit = fitype(['offset+Amp*cos(x/period+phi)']);
    curvefit = fit(N0Nrun(:,1),N0Nrun(:,2),cosfit,'StartPoint',
                [0.4,0.4,0.1,0.8], 'Lower',[0.0,0.0,0.8, 2*3.14159], 'Upper',
                [0.5,1.4,2*3.14159]);
    fitvals = coeffvalues(curvefit); %amp,offset,period,phi
    Visibility(t,s,u) = fitvals(1)/fitvals(2);
    y0(t,s,u)=fitvals(2);
    %s_phase(u)=fitvals(4);
    %s_offset(u)=fitvals(2);
    figure(1)
    hold off
    plot(N0Nrun(:,1),N0Nrun(:,2),'LineStyle','none','Marker','o');
    hold on
    plot(curvefit);
    end

    Vis(t,s)=Vis(t,s)*Visibility(t,s,u)/iter;
    Y0(t,s)=Y0(t,s)+y0(t,s,u)/iter;
    VJ(J)=VJ(J)+Visibility(t,s,u)/iter;
    Y0J(J)=Y0J(J)+y0(t,s,u)/iter;
    J=J+1;
end

end

%hold on
%figure(2)
%plot(tauarray,Visibility(:,u,s),'LineStyle','none','Marker','o');
%hold on
%figure(4)
%plot(Arange,y0(:,u,t),'LineStyle','none','Marker','o');
end
118 end
120 for s=1:length(Arange)
121 \texttt{test5(s)=2*pi*5.6*Arange(s)*sin(2*tau*2*pi*5.6);}
122 \texttt{test10(s)=2*pi*5.6*Arange(s)*sin(2*tau*2*pi*5.6);}
123 \end
126 %end
127
128 \texttt{figure(3)}
129 \texttt{plot(test5,Vis(1,:)), 'LineStyle', 'none', 'Marker', 'o')}\texttt{;}
130 \texttt{plot(test10,Vis(2,:)), 'LineStyle', 'none', 'Marker', 'o')}\texttt{;}
131 \texttt{figure(2)}
132 \texttt{plot(splitspeed,Y0J, 'LineStyle', 'none', 'Marker', 'o');}
133 \texttt{figure(3)}
134 \texttt{plot(AJ,VJ, 'LineStyle', 'none', 'Marker', 'o');}
136
function [N,X,P,b] = full3d_int(tau,chi,Amp,t0)

% uses block expansion of plane waves to determine effect of pulses
% uses coherent states to determine time propagation
% going to use capital letters for Bragg beam frame
% and lower case for ho frame

%Each pulse produces more plane wave states so number of pulses to keep
%track off goes like n^4 for order n states considered except only need
%n=-1,0,1 for recombination.

global w Rtheta Rphi x0 p0 M k Xalpha Palpha vb omega hbar

% theta is the angel between z and Z towards Y, phi between y and Y
% x(i)=(x,y,z) and x0 and p0 are initial conditions
vb=zeros(1,3); %will be offset momentum phase in the hamiltonian
w = 2.36e4; % recoil frequency, s^-1
k = 2*pi/780e-9; % wavenumber, m^-1
	hbar = 6.626e-34/2/pi;

theta = 6*pi/180; % polar angel
phi = 1*pi/180; % angel to horizontal towards x

Rtheta = [1 0 0;0 cos(theta) -sin(theta);0 sin(theta) cos(theta)]; %rotation matrix
Rphi   = [cos(phi) -sin(phi) 0;sin(phi) cos(phi) 0;0 0 1]; %makes the rotation matrix

x0=zeros(3,1); %will be initial position and momentum
p0=zeros(3,1);

omega=zeros(1,3); %the trap frequncies
omega(1)=2*pi*5.6;
omega(2)=2*pi*1.1;
omega(3)=2*pi*3.4;

Xalpha=zeros(1,3); %the ho constants for each direction
Xalpha(1)=sqrt(hbar/(2*M*omega(1)));
Xalpha(2)=sqrt(hbar/(2*M*omega(2)));
Xalpha(3)=sqrt(hbar/(2*M*omega(3)));

Palpha=zeros(1,3);
Palpha(1)=sqrt(hbar*M*omega(1)/2);
Palpha(2)=sqrt(hbar*M*omega(2)/2);
Palpha(3)=sqrt(hbar*M*omega(3)/2);

p0(1)=M*Amp(1)*omega(1)*sin(omega(1)*(2*tau-t0(1)));
52  \[ x_0(1) = A \cos(\omega(1) \cdot (2\tau + t_0(1))); \]
53  \[ p_0(2) = M A \omega(2) \sin(\omega(2) \cdot (2\tau - t_0(2))); \]
54  \[ x_0(2) = A \cos(\omega(2) \cdot (2\tau + t_0(2))); \]
55  \[ p_0(2) = M A \omega(2); \] \& \( A \) is actually a velocity now
56  \[ x_0(2) = 0; \]
57  \[ p_0(3) = M A \omega(3) \sin(\omega(3) \cdot (2\tau - t_0(3))); \]
58  \[ x_0(3) = A \cos(\omega(3) \cdot (2\tau + t_0(3))); \] \( A \omega(3) \cos(\omega(3) \cdot 2\tau); \)
59
60  \[ a = \text{zeros}(3,3,3,3); \]
61  \% Last index labels the operations, the first four
62  \% Label the amplitudes after the operations. For example, after the reflect
63  \% we may have some atoms in a +1 (index=3) that were initially in the -1
64  \% (index=1) after the split. That information would be in a(2,1,3,2,2)
65  \% Where the last two indices are 2 even though they don't matter. After
66  \% recombination these atoms might be in a moving state +1 after reflecting
67  \% to -1 then, that would be in a(4,1,3,1,1)
68
69  \[ a(2,2,2,2,1) = 1; \] \% Initial plane wave state
70
71  \[ \text{gamma} = \text{zeros}(3,3,3,3); \] \% Phase for each split, reflect1, reflect2 wave and direction
72
73  \% For the following arrays, \((i,j,k,l,m)\) \(i\) is the coordinate, \(j\) corresponds
74  \% to the split waves \((j=1,2,3\) goes with \(n=-1,0,1)\). \(k\) goes with reflect1. \(l\)
75  \% with reflect 2 and \(m\) with recombination
76
77  \[ \text{alpha} = \text{zeros}(3,3,3,3); \] \% Ho states values
78
79  \[ x = \text{zeros}(3,3,3,3,3); \] \% Positions for all states in prime frame (beam)
80  \[ p = \text{zeros}(3,3,3,3,3); \] \% Momentum for all states in prime frame (beam)
81
82  \[ x = \text{zeros}(3,3,3,3,3); \] \% Positions for all states in unprime frame (ho)
83  \[ p = \text{zeros}(3,3,3,3,3); \] \% Momentum for all states in unprime frame (ho)
84
85
86  \% Load initial phase space into master phase space array
87
88  \% For \(u=1:3\)
89  \[ x(u,2,2,2,1) = x0(u); \]
90  \[ p(u,2,2,2,1) = p0(u); \]
91
92  \% Rotate to beam frame
93
94  \% For \(u=1:3\)
95  \[ X(:,u,2,2,2,1) = \text{Rotate}(x(:,u,2,2,2,1),1); \]
96  \[ P(:,u,2,2,2,1) = \text{Rotate}(p(:,u,2,2,2,1),1); \]
103     end
104
105  [a,X,P,gamma]=split(a,X,P,gamma);
106  [a,X,P,gamma]=free_evolve_3d_ho(tau,a,X,P,gamma,1);
107  [a,X,P]=reflect1_ho(a,X,P);
108  [a,X,P,gamma]=free_evolve_3d_ho(2*tau,a,X,P,gamma,2);
109  [a,X,P]=reflect2_ho(a,X,P);
110  [a,X,P,gamma]=free_evolve_3d_ho(tau,a,X,P,gamma,3);
111  a=applyphase_ho(a,chi);
112  [a,X,P]=recombination_ho(a,X,P);
113  N=overlap(a,X);
114  %construct final amplitudes
115
116  %Nminu=(abs(a(2,2,2,1,4)+a(1,3,1,1,4)+a(3,2,1,1,4)+a(3,1,3,1,4)+a(1,2,3,1,4))^2;
117  %Nzero=(abs(a(2,2,2,2,4)+a(1,3,1,2,4)+a(3,2,1,2,4)+a(3,1,3,2,4)+a(1,2,3,2,4))^2;
118  %Nplus=(abs(a(2,2,2,3,4)+a(1,3,1,3,4)+a(3,2,1,3,4)+a(3,1,3,3,4)+a(1,2,3,3,4))^2;
119
120  %N0_N=Nzero/(Nplus+Nzero+Nminu);
121  %Np_Nm_N=(Nplus-Nminu)/(Nplus+Nminu);
122
123  %want to know amp squared in each state for making plots and checks
124
125  b=zeros(3,3,3,3,4);
126  for u=1:3
127      for v=1:3
128        for s=1:3
129          for r=1:3
130            for q=1:4
131                b(u,v,s,r,q)=abs(a(u,v,s,r,q))^2;
132            end
133          end
134        end
135      end
136  end
137
138  end
139
140
141
142
function x2 = Rotate(x1,direction)

global Rtheta Rphi

if direction==1
    x2=Rtheta*Rphi*x1; %rotate to prime frame (beam coords)
else
    x2=inv(Rphi)*inv(Rtheta)*x1; %rotate back from prime frame (ho coords)
end
end
function dydt = bragg_deriv(t,y)

global w beta vb;

% n=0,1,-1 = y1,y2,y3
h = [w*(vb(1)/(4*w)-2)^2     beta/2         0;
    beta/2           w*(vb(2)/(4*w))^2   beta/2;
    0                    beta/2    w*(vb(3)/(4*w)+2)^2];

% n=-4,-3,-2,-1,0,1,2,3,4 = y1..y9

% the hamiltonian below can handle many more momentum states
% makes the program take to long to be really useful on a normal machine

h =[w*(d/(4*w)-8)^2  b/2     0        0      0     0       0        0        0;
    0         b/2    w*(d/(4*w)-6)^2   b/2       0      0       0        0        0;
    0         0         b/2    w*(d/(4*w)-4)^2  b/2       0      0        0        0;
    0         0         0         b/2    w*(d/(4*w)-2)^2  b/2       0      0        0;
    0         0         0         0         b/2     w*(d/(4*w)+0)^2     b/2  0      0;
    0         0         0         0         0         b/2     w*(d/(4*w)+2)^2   b/2  0;
    0         0         0         0         0         0         b/2     w*(d/(4*w)+4)^2  b/2;  
    0         0         0         0         0         0         0         b/2     w*(d/(4*w)+6)^2  b/2;
    0         0         0         0         0         0         0         0         b/2     w*(d/(4*w)+8)^2];

dydt = -i*h*y;
function [aout,Xout,Pout,gammaout] = split(ain,Xin,Pin,gammain)

global w beta vb M k hbar

% w is recoil frequency
% d = 2 k v_0 for initial atom velocity v_0
% beta is coupling amplitude, from V = hbar beta cos(2kz)

% first transfer all old info to new arrays, modify elements as needed
Pout=Pin;
aout=ain;
a1=ain;
a2=ain;
Xout=Xin;
gammaout=gammain;

beta = 2*sqrt(2)*w;

vb(1) = 2*k*Pin(2,2,2,2,2,1)/M;
vb(2) = 2*k*Pin(2,2,2,2,2,1)/M;
vb(3) = 2*k*Pin(2,2,2,2,2,1)/M;  % speed offset phase for split operation

ts = pi/4/sqrt(2)/w;  % split pulse time
tw = pi/4/w;  % wait pulse time

yin(1)=ain(1,2,2,2,1);
yin(2)=ain(2,2,2,2,1);
yin(3)=ain(3,2,2,2,1);

% Splitting pulses:
[t,y] = ode45(@bragg_deriv,[0 ts],yin);
yout = y(length(y),:);
y1=free_evolve(tw,yout);

[t,y] = ode45(@bragg_deriv,[0 ts],y1);
yout = y(length(y),:);
aout(1,2,2,2,1)=yout(1);
aout(2,2,2,2,1)=yout(2);
aout(3,2,2,2,1)=yout(3);

% change momentum
% non-moving state doesn't change here is the minus state
Pout(1,1,2,2,2,1)=Pin(1,2,2,2,2,1);
Pout(2,1,2,2,2,1)=Pin(2,2,2,2,2,1)-2*hbar*k;
Pout(3,1,2,2,2,1)=Pin(3,2,2,2,2,1);

% and the plus state

52 \text{Pout}(1,3,2,2,2,1)=\text{Pin}(1,2,2,2,2,1);
53 \text{Pout}(2,3,2,2,2,1)=\text{Pin}(2,2,2,2,2,1)+2\ast\text{hbar}\ast k;
54 \text{Pout}(3,3,2,2,2,1)=\text{Pin}(3,2,2,2,2,1);
55
56 \text{% also copy positions over}
57 \text{Xout}(1,1,2,2,2,1)=\text{Xin}(1,2,2,2,2,1);
58 \text{Xout}(2,1,2,2,2,1)=\text{Xin}(2,2,2,2,2,1);
59 \text{Xout}(3,1,2,2,2,1)=\text{Xin}(3,2,2,2,2,1);
60
61 \text{Xout}(1,3,2,2,2,1)=\text{Xin}(1,2,2,2,2,1);
62 \text{Xout}(2,3,2,2,2,1)=\text{Xin}(2,2,2,2,2,1);
63 \text{Xout}(3,3,2,2,2,1)=\text{Xin}(3,2,2,2,2,1);
64 \text{end}
65
66
function [aout,Xout,Pout] = reflect1_ho(ain,Xin,Pin)
% n is the split state label -1->1 0->2 1->3
global w beta M k hbar vb
% w is recoil frequency
% d = 2 k v_0 for initial atom velocity v_0
% b is coupling amplitude, from V = hbar b cos(2kz)
beta = 2*sqrt(6)*w*(1.000);
Pout=Pin;
aout=ain;
Xout=Xin;
tr = pi/2/w;  \% reflect pulse time
for n=1:3
    vb=ones(1,3)*hbar*k*(Pin(2,n,2,2,2,1)-2*(n-2)*hbar*k)/M;
    \%vb=vb
    yin=zeros(1,3);
    yin(n)=ain(n,2,2,2,1);
    \[t,y\] = ode45(@bragg_deriv,[0 tr],yin);
    \%writes new y momentum
    Pout(2,n,1,2,2,2)=Pin(2,n,2,2,2,1)-2*(n-1)*hbar*k;
Pout(2,n,2,2,2)=Pin(2,n,2,2,2,1)-2*(n-2)*hbar*k;
Pout(2,n,3,2,2,2)=Pin(2,n,2,2,2,1)-2*(n-3)*hbar*k;
    \%write new x and z momentum and position
    Pout(1,n,1,2,2,2)=Pin(1,n,2,2,2,1);
Pout(1,n,2,2,2,2)=Pin(1,n,2,2,2,1);
Pout(1,n,3,2,2,2)=Pin(1,n,2,2,2,1);
Pout(3,n,1,2,2,2)=Pin(3,n,2,2,2,1);
Pout(3,n,2,2,2,2)=Pin(3,n,2,2,2,1);
Pout(3,n,3,2,2,2)=Pin(3,n,2,2,2,1);
Xout(1,n,1,2,2,2)=Xin(1,n,2,2,2,1);
Xout(1,n,2,2,2,2)=Xin(1,n,2,2,2,1);
Xout(1,n,3,2,2,2)=Xin(1,n,2,2,2,1);
Xout(2,n,1,2,2,2)=Xin(2,n,2,2,2,1);
Xout(2,n,2,2,2,2)=Xin(2,n,2,2,2,1);
Xout(2,n,3,2,2,2)=Xin(2,n,2,2,2,1);
Xout(3,n,1,2,2,2)=Xin(3,n,2,2,2,1);
Xout(3,n,2,2,2,2)=Xin(3,n,2,2,2,1);
Xout(3,n,3,2,2,2)=Xin(3,n,2,2,2,1);
aout(n,1,2,2,2)=yout(1);
aout(n,2,2,2,2)=yout(2);
aout(n,3,2,2,2)=yout(3);
end
end
function [aout,Xout,Pout] = reflect2_ho(ain,Xin,Pin)
% n is the split state label -1->1 0->2 1->3
global w beta M k hbar vb
% w is recoil frequency
% d = 2 k v_0 for initial atom velocity v_0
% b is coupling amplitude, from V = hbar b cos(2kz)
7 beta = 2*sqrt(6)*w*(1.000);
8 Pout=Pin;
9 Xout=Xin;
10 aout=ain;
11 tr = pi/2/w;  % reflect pulse time
12 for u=1:3
13     for v=1:3
14         yin=zeros(1,3);
15         vb=ones(1,3)*2*k*(Pin(2,u,v,2,2,2)-2*(v-2)*hbar*k)/M;  % offset momentum phase
16         yin(v)=ain(u,v,2,2,2);  % input amplitudes
17         %vb(v)=;
18         [t,y] = ode45(@bragg_deriv,[0 tr],yin);
19         yout = y(length(y),:);
20         aout(u,v,1,2,3)=yout(1);
21         aout(u,v,2,2,3)=yout(2);
22         aout(u,v,3,2,3)=yout(3);
23         % new y momentum
24         Pout(2,u,v,1,2,3)=Pin(2,u,v,2,2,2)-2*hbar*k*(v-1);
25         Pout(2,u,v,2,2,3)=Pin(2,u,v,2,2,2)-2*hbar*k*(v-2);  % the zero order path
26         Pout(2,u,v,3,2,3)=Pin(2,u,v,2,2,2)-2*hbar*k*(v-3);
27         % the new x and z momentum and position
28         Pout(1,u,v,1,2,3)=Pin(1,u,v,2,2,2);
29         Pout(1,u,v,2,2,3)=Pin(1,u,v,2,2,2);
30         Pout(1,u,v,3,2,3)=Pin(1,u,v,2,2,2);
31         Pout(3,u,v,1,2,3)=Pin(3,u,v,2,2,2);
32         Pout(3,u,v,2,2,3)=Pin(3,u,v,2,2,2);
33         Pout(3,u,v,3,2,3)=Pin(3,u,v,2,2,2);
34         Xout(1,u,v,1,2,3)=Xin(1,u,v,2,2,2);
35         Xout(1,u,v,2,2,3)=Xin(1,u,v,2,2,2);
36         Xout(1,u,v,3,2,3)=Xin(1,u,v,2,2,2);
37         Xout(2,u,v,1,2,3)=Xin(2,u,v,2,2,2);
38         Xout(2,u,v,2,2,3)=Xin(2,u,v,2,2,2);
39         Xout(2,u,v,3,2,3)=Xin(2,u,v,2,2,2);
40         Xout(3,u,v,1,2,3)=Xin(3,u,v,2,2,2);
41         Xout(3,u,v,2,2,3)=Xin(3,u,v,2,2,2);
42         Xout(3,u,v,3,2,3)=Xin(3,u,v,2,2,2);
43         Xout(3,u,v,1,2,3)=Xin(3,u,v,2,2,2);
44         Xout(3,u,v,2,2,3)=Xin(3,u,v,2,2,2);
45         Xout(3,u,v,3,2,3)=Xin(3,u,v,2,2,2);
46         Xout(3,u,v,1,2,3)=Xin(3,u,v,2,2,2);
47         Xout(3,u,v,2,2,3)=Xin(3,u,v,2,2,2);
48     end
49 end
50 end
function aout = applyphase_ho(ain, chi)
aout = ain;
for u=1:3
  for v=1:3
    for s=1:3
      aout(u,v,s,2,3) = ain(u,v,s,2,3) * exp(1i*(s-2)*chi/2);
    end
  end
end
end
function [aout,Xout,Pout] = recombination_ho(ain,Xin,Pin)

global w beta vb M k hbar

% w is recoil frequency
% d = 2 k v_0 for initial atom velocity v_0
% beta is coupling amplitude, from V = hbar b cos(2kz)
% going to calculate 15 amplitudes
% at this point I don't care about P or X
Pout=Pin;
Xout=Xin;
%P1=zeros(3,3,3,3,3,4);
aout=ain;
a1=ain;
a2=ain;
beta = 2*sqrt(2)*w;

for u=1:3 %the split state
    for v=1:3 %the reflect state
        for s=1:3 %the 2nd reflect

            vb=ones(1,3)*2*k*(Pin(2,u,v,s,2,3)-2*hbar*k*(s-2))/M;

            ts = pi/4/sqrt(2)/w; % split pulse time
            tw = pi/4/w; % wait pulse time
            yin=zeros(1,3);
            yin(s)=ain(u,v,s,2,3);

            [t,y] = ode45(@bragg_deriv,[0 ts],yin);
            yout = y(length(y),:);
            %returns the split state amplitudes in yout

            y1=free_evolve(tw,yout);
            %evolves everything
            [t,y] = ode45(@bragg_deriv,[0 ts],y1);

            yout = y(length(y),:);

            aout(u,v,s,1,4)=yout(1);
            aout(u,v,s,2,4)=yout(2); %writes the amplitudes
            aout(u,v,s,3,4)=yout(3);

            Pout(2,u,v,s,1,4)=Pin(2,u,v,s,2,3)-2*hbar*k*(s-2);
            Pout(2,u,v,s,2,4)=Pin(2,u,v,s,2,3)-2*hbar*k*(s-3);
            Pout(2,u,v,s,3,4)=Pin(2,u,v,s,2,3)-2*hbar*k*(s-1);

        end
    end
end

Pout(1,u,v,s,1,4)=Pin(1,u,v,s,2,3);
Pout(1,u,v,s,2,4)=Pin(1,u,v,s,2,3);
Pout(1,u,v,s,3,4)=Pin(1,u,v,s,2,3);
Pout(3,u,v,s,1,4)=Pin(3,u,v,s,2,3);
Pout(3,u,v,s,2,4)=Pin(3,u,v,s,2,3);
Pout(3,u,v,s,3,4)=Pin(3,u,v,s,2,3);

Xout(1,u,v,s,1,4)=Xin(1,u,v,s,2,3);
Xout(1,u,v,s,2,4)=Xin(1,u,v,s,2,3);
Xout(1,u,v,s,3,4)=Xin(1,u,v,s,2,3);

Xout(2,u,v,s,1,4)=Xin(2,u,v,s,2,3);
Xout(2,u,v,s,2,4)=Xin(2,u,v,s,2,3);
Xout(2,u,v,s,3,4)=Xin(2,u,v,s,2,3);

Xout(3,u,v,s,1,4)=Xin(3,u,v,s,2,3);
Xout(3,u,v,s,2,4)=Xin(3,u,v,s,2,3);
Xout(3,u,v,s,3,4)=Xin(3,u,v,s,2,3);

end
end
end
function [Nout] = overlap(ain,Xin)
global hbar M omega

% look at three clouds the zero and the two main ones
% find their positions and the average (center of mass)
% do it in coordinates of the TF lengths
Nout=zeros(3,1);
mu=2*pi*hbar*15;
L=zeros(3,1);
r=zeros(3,3);
Rcom=zeros(3,1);

% consider only the two main and zero loops and their resulting split
% results
for j=1:3
    L(j)=sqrt(2*mu/M/omega(j)^2);
    r(j,1)=Xin(j,1,1,1,2)/L(j);
    r(j,2)=Xin(j,2,2,2,2)/L(j);
    r(j,3)=Xin(j,3,2,3,3)/L(j);
    Rcom(j)=(r(j,1)+r(j,2)+r(j,3))/3;
end

% create a grid and for every point see how many spheres it is in then use
% those spheres to calculate a local probability for what direction it goes
% then add up all grid points probabilities. For first run just assume a
% uniform sphere
step=0.05; % grid step
width=1.3;
Xg = (Rcom(1)-width:step:Rcom(1)+width);
Yg = (Rcom(2)-width:step:Rcom(2)+width);
Zg = (Rcom(3)-width:step:Rcom(3)+width);

Nminu=(abs(a(2,2,2,1,4)+a(1,3,1,1,4)+a(3,2,1,1,4)+a(3,1,3,1,4)+a(1,2,3,1,4)))^2;
Nzero=(abs(a(2,2,2,2,4)+a(1,3,1,2,4)+a(3,2,1,2,4)+a(3,1,3,2,4)+a(1,2,3,2,4)))^2;
Nplus=(abs(a(2,2,2,3,4)+a(1,3,1,3,4)+a(3,2,1,3,4)+a(3,1,3,3,4)+a(1,2,3,3,4)))^2;

Amp=zeros(3,1);
Nout=zeros(3,1);

for u=1:length(Xg)
    for v=1:length(Yg)
        for s=1:length(Zg)
            % figure out if any of minu (before recomb) cloud is at this point
            rprime=sqrt((Xg(u)-r(1,1))^2+(Yg(v)-r(2,1))^2+(Zg(s)-r(3,1))^2);
            if rprime <= 1
                % add its amplitude to the overall amplitudes for the final
                % directions
                Amp(1)=Amp(1)+ain(1,3,1,1,4);
                Amp(2)=Amp(2)+ain(1,3,1,2,4);
                Amp(3)=Amp(3)+ain(1,3,1,3,4);
            end
        end
    end
end

Amp(1)=Amp(1)+ain(1,3,1,1,4);
Amp(2)=Amp(2)+ain(1,3,1,2,4);
Amp(3)=Amp(3)+ain(1,3,1,3,4);
% figure out if any of zero cloud is here
rprime = sqrt((Xg(u) - r(1,2))^2 + (Yg(v) - r(2,2))^2 + (Zg(s) - r(3,2))^2);
if rprime <= 1
    % add its amplitude to the overall amplitudes for the final direction
    Amp(1) = Amp(1) + ain(2,2,1,4);
    Amp(2) = Amp(2) + ain(2,2,2,4);
    Amp(3) = Amp(3) + ain(2,2,3,4);
end

% figure out if any of plus cloud is here
rprime = sqrt((Xg(u) - r(1,3))^2 + (Yg(v) - r(2,3))^2 + (Zg(s) - r(3,3))^2);
if rprime <= 1
    % add its amplitude to the overall amplitudes for the final direction
    Amp(1) = Amp(1) + ain(3,1,3,1,4);
    Amp(2) = Amp(2) + ain(3,1,3,2,4);
    Amp(3) = Amp(3) + ain(3,1,3,3,4);
end
% calculate the number of atoms there and then add to total
for n = 1:3
    Nout(n) = Nout(n) + abs(Amp(n))^2*step^3;
end
Nout = Nout / (Nout(1) + Nout(2) + Nout(3)); % normalize
Appendix D

ANSYS code

This is the code stored in a “.mac” file that runs in the program called ANSYS to do a finite element analysis of the conductors in the polarizability experiment. The code determines the electric field in 2 directions, $x$ and $y$, which translate $y$ and $z$ respectively in the experiment. The comments after the # sign must be taken out for it run. Multiply the output by $t = 151.28 \, \mu m$ to normalize it to $E_0 = V_0/t$. This was written by Robert Horne.

/PREP7  # units are in µm
GAP = 40  # gap between the split conductor
SEP = 151.28  # etalon spacing
LENGTH = 1000  # width of the conductor
MESH1 = 500  # describes how fine of pieces to split the conductor into
MESH2 = 10
RECTNG, GAP/2,GAP/2+LENGTH,0,SEP/2  # this draws the conductor geometry
RECTNG, -GAP/2,-GAP/2-LENGTH,0,SEP/2
RECTNG, 0,GAP/2+LENGTH,0,-SEP/2
RECTNG, 0,-GAP/2-LENGTH,0,-SEP/2
RECTNG,-3*LENGTH,3*LENGTH,-LENGTH-SEP/2,LENGTH+SPACE/2
ASBA, 5,1,,DELETE,KEEP
ASBA, 6,2,,DELETE,KEEP
ASBA, 5,3,,DELETE,KEEP
ASBA, 6,4,,DELETE,KEEP
AGLUE, 1,2,3,4,5
NUMCMP, AREA
FINISH
/PREP7
LESIZE, 7,,MESH1,1,,1
LESIZE, 3,,MESH1,1,,1
LESIZE, 9,,MESH1,1,,1
LESIZE, 25,,MESH1,1,,1
LESIZE, 5,,MESH1,1,,1
LESIZE, 1,,MESH1,1,,,1
LESIZE, 21,,MESH2,1,,,1
LESIZE, 24,,MESH2,1,,,1
SMRTSIZE, 3
AMESH, 1,5,1
FINISH
/SOLUTION    # one volt used
DL, 3,,VOLT,1,1
DL, 7,,VOLT,0,1
DL, 9,,VOLT,0,1
DL, 25,,VOLT,0,1
DL, 17,,VOLT,0,1
DL, 18,,VOLT,0,1
DL, 19,,VOLT,0,1
DL, 20,,VOLT,0,1
SOLVE
FINISH
/POST1    # compute the fields on lines 3 lines of constant y (z in the exp)
PATH, path1, 2,30,1000
PPATH, 1,,500,0,0    # the middle of the conductor
PPATH, 2,,500,0,0
PDEF, volt,VOLT,,AVG
PDEF, efx,EF,X,AVG
PDEF, efy,EF,Y,AVG
PATH, path2, 2,30,1000
PPATH, 1,,500,10,0    # the middle +10
PPATH, 2,,500,10,0
PDEF, volt,VOLT,,AVG
PDEF, efx,EF,X,AVG
PDEF, efy,EF,Y,AVG
PATH, path3, 2,30,1000
PPATH, 1,,500,-10,0    # the middle -10
PPATH, 2,,500,-10,0
PDEF, volt,VOLT,,AVG
PDEF, efx,EF,X,AVG
PDEF, efy,EF,Y,AVG
Appendix E

Current Noise

Current noise in the magnetic traps can lead to atom loss and heating, residual motion in the guide and interferometer phase noise [58]. Figures E.1 and E.2 figures show MHz current noise on the spherical quadrupole current. MHz noise flip the spin of the atoms in the same as evaporative cooling. The capacitance referred to in the first figure is a polypropylene capacitor placed in parallel with the magnet coils, which should tend to short out high frequency noise. The second figure demonstrates that much of the noise on the magnet coils is sourced from the power supply’s control source.

Figure E.3 displays the current noise on the new external wave guide conductors. This noise can cause phase noise on the interferometer. A Comparison of the power spectrum, $S(\nu)$, to that in [58] shows that the noise on the new waveguide is approximately the same as the old one. This makes sense because they share essentially the same power source. Also shown is that there are only very small sidebands at the 10 Hz level.
Figure E.1: Quadrupole Current Noise and Capacitance

Full current noise based on current control mechanism with no switching circuit and 500 nF Capacitor across coils

Figure E.2: Quadrupole Current Noise by Control Source
Figure E.3: New Waveguide Current Noise Spectrum
Bibliography


